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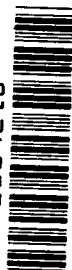
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LATERAL AERODYNAMIC PARAMETERS  
EXTRACTED FROM FLIGHT DATA  
FOR THE F-8C AIRPLANE  
IN MANEUVERING FLIGHT

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LATERAL AERODYNAMIC PARAMETERS EXTRACTED  
FROM FLIGHT DATA FOR THE F-8C AIRPLANE  
IN MANEUVERING FLIGHT

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SUMMARY

Flight-test data have been used to extract the lateral aerodynamic parameters of the F-8C airplane at moderate to high angles of attack. The data were obtained during perturbations of the aircraft from steady turns with trim normal accelerations from 1.5g to 3.0g. The angle-of-attack variation from trim was negligible.

Although wind-tunnel data indicate that the rolling and yawing moments are somewhat nonlinear with angle of attack, the angle-of-attack variations are small; therefore, the linear aerodynamic coefficients extracted from the flight tests permit computation of motion time histories which are in close agreement with the measured time histories. The aerodynamic coefficients extracted from flight data were compared with several other sets of coefficients, and the extracted coefficients resulted in characteristics for the Dutch roll mode (at the highest angles of attack) similar to those of a set of coefficients that has been the basis of several simulations of the F-8C.

INTRODUCTION

The National Aeronautics and Space Administration (NASA) is currently involved in research on fly-by-wire control systems for aircraft. A discussion of the NASA fly-by-wire program is given in reference 1. The aircraft currently used by NASA as a test bed to study digital fly-by-wire systems is an F-8C airplane with a standard airframe. Previously determined aerodynamics of the subject airplane came primarily from wind-tunnel tests and analytical calculations, and the mathematical aerodynamic model of the airplane was considered to be reasonable, especially for trimmed level flight. To substantiate the existing linear aerodynamic model for the F-8C at moderate to high angles of attack, some flight tests were made with the angle of attack as high as  $16^{\circ}$ .

A maximum-likelihood extraction procedure was used to analyze the flight data. In this procedure, a set of equations of motion is used to calculate aircraft response to specified control inputs. Initial estimates of aerodynamic parameters (either from theory or

from wind-tunnel tests) are used for the initial motion computations. An iterative digital computer program then selects a set of aerodynamic parameters that minimizes the difference between the computed profiles and the flight profiles. This program has been used to determine the aerodynamic parameters for several aircraft in the 1g trimmed-flight condition. (See refs. 2 to 4.) The details of the program are contained in reference 5 and in the appendix of this report. The program has not been used previously for lateral flight data taken at moderate to high angles of attack with a trimmed normal acceleration greater than 1g. The program can be used as long as angle-of-attack variations from trim are small, so that the assumption of linear aerodynamics will not be violated. Analytical and wind-tunnel studies have shown that most of the lateral aerodynamic parameters can have non-linear variations with angle of attack over the range used in this investigation and that some of these variations can be significant (ref. 6). For each of the individual flights used in this investigation, the variations in angle of attack from the trim angle of attack were less than  $1^{\circ}$  during 90 percent of the time history and always less than  $2^{\circ}$ . Therefore, a linear model for the aerodynamics was considered adequate to describe the motion of the airplane. The linearity assumptions were successfully used in extraction of the longitudinal aerodynamic parameters for the F-8C at moderate to high angles of attack, and the results are given in reference 7.

The purpose of this paper is to present the lateral aerodynamic parameters of the F-8C airplane as calculated from flight data obtained near trim at Mach numbers of 0.7 and 0.8, with normal accelerations of 1.5g to 3.0g. Also presented are the equations used and additional information on the flight data. These are followed by some comments on the extraction procedure and a discussion of the results of the study.

## SYMBOLS

Values are given in both SI and U.S. Customary Units. The measurements and calculations were made in U.S. Customary Units.

$a_Y$	acceleration measured along Y body axis, g units
b	wing span, m (ft)
$\bar{c}$	wing mean geometric chord, m (ft)
g	acceleration due to gravity, 9.81 m/sec <sup>2</sup> (32.2 ft/sec <sup>2</sup> )
h	altitude, m (ft)

I	moment of inertia, $\text{kg-m}^2$ (slug-ft <sup>2</sup> )
M	Mach number
P	period of oscillatory motion, sec
p	rate of roll, rad/sec or deg/sec
q	rate of pitch, rad/sec or deg/sec
r	rate of yaw, rad/sec or deg/sec
S	wing area, $\text{m}^2$ (ft <sup>2</sup> )
t	time, sec
u	component of velocity along X body axis, m/sec (ft/sec)
V	aircraft total velocity, m/sec (ft/sec)
v	component of velocity along Y body axis, m/sec (ft/sec)
W	aircraft weight, N (lb)
w	component of velocity along Z body axis, m/sec (ft/sec)
X,Y,Z	body coordinate axes through airplane center of gravity
$\alpha$	angle of attack, rad or deg
$\beta$	angle of sideslip, rad or deg
$\delta_a$	aileron deflection (positive for left roll), rad or deg
$\delta_e$	tail-plane deflection (positive for trailing edge down), rad or deg
$\delta_r$	rudder deflection (positive for trailing edge left), rad or deg
$\zeta$	damping ratio

$\theta$	pitch angle, rad or deg
$\rho$	air density, kg/m <sup>3</sup> (slugs/ft <sup>3</sup> )
$\phi$	bank angle, rad or deg

Coefficients and derivatives:

$C_l$  rolling-moment coefficient

$C_n$  yawing-moment coefficient

$C_Y$  side-force coefficient

$$C_{l_p} = \frac{\partial C_l}{\partial \left( \frac{pb}{2V} \right)}$$

$$C_{l_r} = \frac{\partial C_l}{\partial \left( \frac{rb}{2V} \right)}$$

$$C_{l_\beta} = \frac{\partial C_l}{\partial \beta}$$

$$C_{l_{\delta_a}} = \frac{\partial C_l}{\partial \delta_a}$$

$$C_{l_{\delta_r}} = \frac{\partial C_l}{\partial \delta_r}$$

$$C_{n_p} = \frac{\partial C_n}{\partial \left( \frac{pb}{2V} \right)}$$

$$C_{n_r} = \frac{\partial C_n}{\partial \left( \frac{rb}{2V} \right)}$$

$$C_{n\beta} = \frac{\partial C_n}{\partial \beta}$$

$$C_{n\delta_a} = \frac{\partial C_n}{\partial \delta_a}$$

$$C_{n\delta_r} = \frac{\partial C_n}{\partial \delta_r}$$

$$C_{Yp} = \frac{\partial C_Y}{\partial \left( \frac{pb}{2V} \right)}$$

$$C_{Yr} = \frac{\partial C_Y}{\partial \left( \frac{rb}{2V} \right)}$$

$$C_{Y\beta} = \frac{\partial C_Y}{\partial \beta}$$

$$C_{Y\delta_r} = \frac{\partial C_Y}{\partial \delta_r}$$

Subscripts:

c                computed

f                measured in flight

t                trim conditions

X,Y,Z          body coordinate axes through aircraft center of gravity

A dot over a symbol signifies a derivative with respect to time.

## EQUATIONS OF MOTION

The equations of motion used in this study are referred to the body-axis system shown in figure 1 and are as follows:

Y-direction:

$$\begin{aligned} \dot{v} = & pw - ru + g \cos \theta \sin \phi + \frac{1}{2} \rho V^2 S \frac{g}{W} \left[ C_{Y,t} + C_{Y_\beta} (\beta - \beta_t) + C_{Y_p} \frac{pb}{2V} \right. \\ & \left. + C_{Y_r} \frac{rb}{2V} + C_{Y_{\delta_r}} (\delta_r - \delta_{r,t}) \right] \end{aligned} \quad (1)$$

Rolling:

$$\begin{aligned} \dot{p} = & -qr \frac{I_Z - I_Y}{I_X} + (pq + \dot{r}) \frac{I_{XZ}}{I_X} + \frac{1}{2} \frac{\rho V^2 S b}{I_X} \left[ C_{l,t} + C_{l_\beta} (\beta - \beta_t) + C_{l_p} \frac{pb}{2V} \right. \\ & \left. + C_{l_r} \frac{rb}{2V} + C_{l_{\delta_r}} (\delta_r - \delta_{r,t}) + C_{l_{\delta_a}} (\delta_a - \delta_{a,t}) \right] \end{aligned} \quad (2)$$

Yawing:

$$\begin{aligned} \dot{r} = & -pq \frac{I_Y - I_X}{I_Z} - (qr - \dot{p}) \frac{I_{XZ}}{I_Z} + \frac{1}{2} \frac{\rho V^2 S b}{I_Z} \left[ C_{n,t} + C_{n_\beta} (\beta - \beta_t) + C_{n_p} \frac{pb}{2V} \right. \\ & \left. + C_{n_r} \frac{rb}{2V} + C_{n_{\delta_a}} (\delta_a - \delta_{a,t}) + C_{n_{\delta_r}} (\delta_r - \delta_{r,t}) \right] \end{aligned} \quad (3)$$

Auxiliary equations:

$$\alpha = \tan^{-1} \frac{w}{u}$$

$$\beta = \sin^{-1} \frac{v}{V}$$

$$a_Y = \frac{1}{g} (\dot{v} + ur - wp - g \cos \theta \sin \phi)$$



$$\dot{\phi} = p + (q \sin \phi + r \cos \phi) \tan \theta$$

The three-degree-of-freedom equations (eqs. (1) to (3)) were solved for the lateral motions. The longitudinal variables  $u$ ,  $w$ ,  $q$ , and  $\theta$  used in these equations were the flight-measured values, and hence the nonlinear contributions of these terms were used in the equations.

## FLIGHT DATA

### Description of Airplane

The airplane used, a modified prototype F-8C, has been a flight-test vehicle since its manufacture in 1958. The F-8C is a single-seat, high-performance airplane with a single jet engine embedded in the fuselage. Pitch control is achieved with a unit horizontal tail. The center of gravity was at 29.0 percent of the mean geometric chord. The  $X$  body axis was parallel to and 10.16 cm (4 in.) above water line 100. (See fig. 1.) The geometric characteristics of the airplane are given in table I.

### Flight Tests

The data used in this report were obtained from flights made at the Hugh L. Dryden Flight Research Center, and are shown in figures 2 to 6. The pilot was instructed to fly a coordinated turn at nominal Mach numbers of 0.7 and 0.8 with nominal trim angles of attack of  $9^\circ$  and  $13^\circ$ . The worst case of angle-of-attack variation is shown in figure 3, where, as can be seen, the angle of attack varied from the trim angle of attack by less than  $2^\circ$ . A  $1^\circ$  variation in  $\alpha$  represents a 0.2g variation in normal acceleration. The airplane stability augmentation systems were off during the tests. The actual test conditions for each individual run are given in table II. The airplane mass and moments of inertia listed in table II were calculated as a function of the percent of fuel in the airplane and were obtained from tables supplied by the Dryden Flight Research Center. The mass and moments of inertia used are average values for the test duration. Since the mass varied less than 3 percent and the moments of inertia by less than 1 percent, these variations were not accounted for in the parameter estimation.

Pertinent data recorded during the flight tests included lateral acceleration  $a_Y$ ; the difference between total pressure and static pressure measured on a nose boom extending 1.83 m (6 ft) in front of the airplane; pitch attitude  $\theta$ ; bank angle  $\phi$ ; pitch rate  $q$ ; yaw rate  $r$ ; roll rate  $p$ ; indicated angle of attack  $\alpha$  and indicated angle of sideslip  $\beta$  measured by vanes on the nose boom; pressure altitude; control surface positions (aileron  $\delta_a$  and rudder  $\delta_r$ ); and time  $t$ . The full-scale range of the instruments is given in table III. All the data were recorded on magnetic tape by an onboard recorder using a

pulse-code-modulation (PCM) recording system. Additional information on the data acquisition system can be found in reference 6.

### Data Preparation

The data were initially recorded, digitized, and calibrated at Dryden Flight Research Center. A digital tape with the data in engineering units was sent to Langley Research Center. The acceleration data were corrected for instrument location. The difference in total pressure and static pressure measurements was assumed to be the dynamic pressure. Density was determined from the standard atmosphere tables for the measured pressure altitude and the airspeed was calculated from dynamic pressure. The indicated angles of attack and sideslip were corrected for the effects of aircraft angular rates. The linear velocities along the vehicle body axes were calculated from the airspeed, angle of attack, and angle of sideslip. All data were recorded on tape at the rate of 20 points per second. The tapes were then ready for use in the extraction program.

## RESULTS AND DISCUSSION

Data for the flight conditions listed in table II were used with the mathematical model given in order to determine iteratively a set of aerodynamic derivatives for each of the flight conditions. The measured and computed time histories for each flight condition are shown in figures 2 to 6, with the measured data represented by dotted lines. The computed time histories shown are those attained after the differences between the measured and calculated time histories became small. The difference was considered small whenever

$$\frac{|R|_i - |R|_{i+1}}{|R|_i} < 0.1,$$
 where  $R$  is the estimate of the error covariance matrix, as defined

in the appendix, and  $i$  refers to the  $i$ th iteration. This inequality will be referred to as the cost function criterion. The figures show that in all tests, the computed time histories were in close agreement with the flight-record time histories. It should also be noted that the parameter values determined were consistent from run to run. Table IV gives the standard deviations of the computed states from the measured states at convergence. The inverse square of each quantity in table IV was used as a diagonal term in the weighting matrix to obtain the fit of computed data to flight data shown in figures 2 to 6. The standard deviation of each fit can be seen to be less than 3 percent of the full-scale instrument range, which was assumed to be the uncertainty in the measured data (see table III). For some quantities, the standard deviation was less than 1 percent of the full-scale measured quantity.

The derivatives extracted for each flight condition (the derivatives which resulted in the computed time histories of figs. 2 to 6) are listed in table V along with their estimated standard deviations (Cramér-Rao lower bound). It should be noted that for  $M = 0.81$

and  $\alpha_t = 8.2^\circ$ , a single set of numbers is given which fits the data for both aileron input and rudder input. The single set was determined by using the values from the two runs that were least affected by correlation and that had the smallest standard deviations. As can be seen, this single set gave a good fit to all the data taken at  $M = 0.81$  with  $\alpha_t = 8.2^\circ$ . If the estimated standard deviations of the parameters were less than 10 percent of the values extracted, confidence in the values obtained was indicated. The parameters which were not considered well determined were  $C_{Y_p}$ ,  $C_{Y_r}$ ,  $C_{Y_{\delta_r}}$ , and  $C_{n_{\delta_a}}$ .

The derivatives which were not considered well determined do not usually have a strong effect on the motion of the airframe. Therefore, it is difficult to excite the airframe so that there will be sufficient information in the data for a confident extraction of these derivatives.

When analyzing the extraction results, the effects of correlation must also be considered. If correlations between parameters are high, erroneous parameter values can

result, and the inversion of the matrix  $\left[ \sum_{i=1}^N M(t_i)^T R^{-1} M(t_i) \right]$  (defined in the appendix)

can be affected. The most obvious effects of correlation are two parameters assuming erroneous values but the fit to the data remaining good, or the cost function oscillating or

diverging because the matrix  $\left[ \sum_{i=1}^N M(t_i)^T R^{-1} M(t_i) \right]$  is nearly singular (refs. 2 and 8).

The correlation matrices for the aerodynamic parameters extracted are shown in tables VI(a) to VI(e). Although some correlations were high, none of the extracted parameter values seemed unreasonable, and convergence problems were encountered in only one run.

For the run in which convergence problems occurred, a procedure discussed in reference 2 was initially used to determine parameter values. This procedure first examined the covariance matrix to determine which parameters were potentially correlated. The covariance matrix for the run with  $\alpha_t = 13^\circ$  and  $M = 0.66$  indicated that  $C_{l_{\delta_a}}$  was correlated with  $C_{l_\beta}$  and  $C_{l_p}$  and that  $C_{n_{\delta_a}}$  was correlated with  $C_{n_\beta}$  and  $C_{n_p}$ . For this particular run, the correlation affected the convergence of the algorithm. To improve the convergence, either  $C_{l_{\delta_a}}$  and  $C_{n_{\delta_r}}$  or  $C_{l_\beta}$ ,  $C_{l_p}$ ,  $C_{n_\beta}$ , and  $C_{n_p}$  were alternately held fixed. Initially  $C_{l_{\delta_a}}$  and  $C_{n_{\delta_r}}$  were held fixed, and  $C_{l_\beta}$ ,  $C_{l_p}$ ,  $C_{n_\beta}$ , and  $C_{n_p}$  were allowed to change for several iterations. Then  $C_{l_\beta}$ ,  $C_{l_p}$ ,  $C_{n_\beta}$ , and  $C_{n_p}$

were fixed, and  $C_{l\delta_a}$  and  $C_{n\delta_r}$  were allowed to vary. This procedure was repeated until the change in the cost function was less than the criterion discussed earlier.

Once a set of numbers was determined to have given a good fit to the data, all the parameters were made active. The fit showed almost no improvement, the parameter values changed less than 5 percent, and the system no longer diverged. Apparently the correlations were not strong enough to affect convergence when the initial guess at the parameter values was close to the converged values. This conclusion is supported by the fact that for the other runs no serious convergence problems were encountered, even though there were correlations for those runs which were as high as those for the run in which convergence problems were initially encountered. The results shown in table VI(b) and in figure 3 are for the final part of the run when all the parameters were active.

For comparison, values of some aerodynamic coefficients obtained from reference 6 are shown in table VII. The numbers from reference 6 are for an altitude of 12.19 km (40 000 ft), were transformed from stability to body axes, and were converted to a center of gravity located at 0.29 $\bar{c}$ . Several of the derivatives that strongly affect the motion of the airframe were calculated using the method of reference 9, and these are also shown in table VII.

Since there were some significant differences between the extracted values and the values from references 6 and 9, an effort was made to determine how well mathematical models using the various sets of parameters represented the airplane. The numbers from reference 6 that are given in table VII were put into the equations of motion, and for the proper Mach number and angle of attack, calculated responses are shown in figures 7 to 9. The fits to the flight data, although not as good as those obtained with the values from table V, are still reasonably good.

In another effort to compare the different sets of parameter values, several of the sets were used to determine the stability characteristics of the mathematical model. These sets are designated A to F, and some of the conditions under which they were generated are given in table VIII. The comparison was made by use of an analysis from reference 10. The reader is cautioned that the approximate analysis given in reference 10 will not work for these cases unless all nonzero trim states are included in the equation of motion when the stability quartic is derived.

The following equations were used in determining the damping ratios and periods of the Dutch roll:

Y-direction:

$$\dot{v} = pw_t - ru_t + g \cos \theta_t (\cos \phi_t) \phi + \frac{1}{2} \rho V_t^2 S \frac{g}{W} \left[ C_{Y,t} + C_{Y_\beta} (\beta - \beta_t) \right. \\ \left. + C_{Y_p} \frac{pb}{2V_t} + C_{Y_r} \frac{rb}{2V_t} + C_{Y_{\delta_r}} (\delta_r - \delta_{r,t}) \right]$$

Rolling:

$$\dot{p} = -rq_t \frac{I_Z - I_Y}{I_X} + (pq_t + \dot{r}) \frac{I_{XZ}}{I_X} + \frac{1}{2} \rho \frac{V_t^2 S b}{I_X} \left[ C_{l,t} + C_{l_\beta} (\beta - \beta_t) \right. \\ \left. + C_{l_p} \frac{pb}{2V_t} + C_{l_r} \frac{rb}{2V_t} + C_{l_{\delta_r}} (\delta_r - \delta_{r,t}) + C_{l_{\delta_a}} (\delta_a - \delta_{a,t}) \right]$$

Yawing:

$$\dot{r} = -pq_t \frac{I_Y - I_X}{I_Z} - (q_t r - \dot{p}) \frac{I_{XZ}}{I_Z} + \frac{1}{2} \rho \frac{V_t^2 S b}{I_Z} \left[ C_{n,t} + C_{n_\beta} (\beta - \beta_t) \right. \\ \left. + C_{n_p} \frac{pb}{2V_t} + C_{n_r} \frac{rb}{2V_t} + C_{n_{\delta_r}} (\delta_r - \delta_{r,t}) + C_{n_{\delta_a}} (\delta_a - \delta_{a,t}) \right]$$

$$\dot{\phi} = p + r \tan \theta_t \cos \phi_t$$

The damping ratios and the periods of the Dutch roll for various mathematical models are given in table VIII. Also given in table VIII are the periods and damping ratios of the Dutch roll mode calculated from the actual flight data (where there was sufficient free oscillation to make the calculations). As expected, when the period and damping ratios were calculated using a mathematical model based on parameter values from reference 6, the damping ratios were less than when the extracted model was used. This conclusion is illustrated by figures 7 to 9. The analysis of the Dutch roll has demonstrated that several sets of parameter values will give a fair fit to a set of flight data and similar stability characteristics;

however, for a given set of data, the extracted mathematical model gives the best fit to the data.

### CONCLUDING REMARKS

Test data obtained during flights of the F-8C airplane at moderate to high angles of attack have been used to determine the lateral aerodynamic parameters of the airplane at four flight conditions. The tests were conducted with the airplane trimmed in a steady turn, with angles of attack approximately  $9^{\circ}$  and  $13^{\circ}$ , and with Mach numbers of 0.7 and 0.8.

The extracted parameters were consistent from run to run and resulted in a fit to the flight data that varied by less than 3 percent of the full instrument range. The parameter values obtained were in fair agreement with values obtained from wind-tunnel and analytical methods. The adequacy of the extracted set of parameter values was further substantiated by showing that other sets of parameter values did not fit the flight data as well as the extracted set. However, period and damping ratios of the Dutch roll modes that were generated by the parameter sets used for comparison were close to those generated by the extracted parameter set.

Analytical and wind-tunnel studies have shown that several of the lateral aerodynamic parameters can vary with angle of attack over the angle-of-attack range tested. However, for each individual flight, the angle-of-attack variation from trim was so small that the linear aerodynamic model used was adequate to describe the motion of the airplane. Therefore, the existing parameter extraction program could be successfully used.

Langley Research Center  
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December 1, 1976

## APPENDIX

### PARAMETER ESTIMATION PROCEDURE

The parameter estimation procedure used in this study is an iterative procedure, developed and discussed in reference 5. The procedure maximizes the conditional likelihood function  $L$  (aerodynamic parameters, weights, and initial conditions):

$$L = \frac{1}{(2\pi)^{1/2} |R|^{1/2}} \exp \left[ -\frac{1}{2} \sum_{i=1}^N (\bar{\mathbf{x}}_{i,f} - \bar{\mathbf{x}}_{i,c})^T R^{-1} (\bar{\mathbf{x}}_{i,f} - \bar{\mathbf{x}}_{i,c}) \right]$$

where  $R$  is the estimate of the error covariance matrix,  $\bar{\mathbf{x}}$  is the vector describing the state of the airplane,  $T$  denotes the transpose, and  $-1$  denotes the inverse. The states used in the likelihood function were  $v$ ,  $p$ ,  $r$ ,  $\phi$ , and  $a_Y$ . The calculated states  $\bar{\mathbf{x}}_{i,c}$  were determined by using the equations of motion previously introduced. In these equations the longitudinal quantities  $u$ ,  $w$ ,  $q$ , and  $\theta$  were input directly into the equations from the flight data tape. The weighting matrix is  $R^{-1}$ , where

$$R = \text{Diagonal} \left[ \sum_{i=1}^N (\bar{\mathbf{x}}_{i,f} - \bar{\mathbf{x}}_{i,c}) (\bar{\mathbf{x}}_{i,f} - \bar{\mathbf{x}}_{i,c})^T \right]$$

The result of maximizing the likelihood function is a parameter-updated equation which is given by

$$\Delta \bar{\mathbf{C}} = \left[ \sum_{i=1}^N \mathbf{M}(t_i)^T R^{-1} \mathbf{M}(t_i) \right]^{-1} \left[ \sum_{i=1}^N \mathbf{M}(t_i)^T R^{-1} (\bar{\mathbf{x}}_{i,f} - \bar{\mathbf{x}}_{i,c}) \right]$$

where  $\bar{\mathbf{C}}$  is the vector of aerodynamic coefficients to be determined,  $\mathbf{M}$  is the matrix of sensitivities of the calculated states with respect to the unknown parameters. (See

ref. 5.) The estimated parameter covariance matrix is  $\left[ \sum_{i=1}^N \mathbf{M}(t_i)^T R^{-1} \mathbf{M}(t_i) \right]^{-1}$ . The

square root of each diagonal element of the covariance matrix (estimated standard deviation) is directly related to the uncertainty in the extracted parameters, and the off-diagonal

## APPENDIX

terms are used to indicate correlations between parameters. The program, whose development is discussed in reference 5, calculates the matrices and vectors required to generate  $\Delta\vec{C}$ . The program then uses  $\Delta\vec{C}$  to change the aerodynamic coefficients iteratively until a fit to a set of flight data is obtained. The steps in the program operation are:

- (1) Choose values for the parameters to be identified.
- (2) Integrate the equations of motion using the current values of the aerodynamic parameters chosen, and get time histories of the states.
- (3) Compute the state covariance matrix  $R$  and the weighting matrix  $R^{-1}$ .
- (4) Calculate the cost function, which is the determinant of  $R$ .
- (5) Integrate the set of differential equations for the sensitivities and then form the matrix  $M$ .
- (6) Form the maximum likelihood estimation equations for the parameter update  $\Delta\vec{C}$ .
- (7) Determine the new parameters using  $\vec{C} = \vec{C}_{\text{current}} + \Delta\vec{C}$ .
- (8) Use the new  $\vec{C}$  as the current value for the next iteration, and continue the process at step (2) until the cost function criterion is satisfied.



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TABLE I.- GEOMETRIC CHARACTERISTICS OF THE F-8C AIRPLANE

Fuselage length, m (ft)	16.52	(54.17)
Wing:		
Area, m <sup>2</sup> (ft <sup>2</sup> )	34.88	(375)
Aspect ratio		3.4
Span, m (ft)	10.88	(35.67)
Mean geometric chord, m (ft)	3.59	(11.78)
Vertical tail:		
Area, m <sup>2</sup> (ft <sup>2</sup> )	10.14	(109)
Aspect ratio		1.5
Span, m (ft)	3.89	(12.75)
Rudder area, m <sup>2</sup> (ft <sup>2</sup> )	1.17	(12.56)
Horizontal tail:		
Area, m <sup>2</sup> (ft <sup>2</sup> )	8.68	(93.4)
Aspect ratio		3.5
Span, m (ft)	5.52	(18.1)
Length (center of gravity to one-fourth of tail mean geometric chord), m (ft)	5.06	(16.6)

TABLE II. - FLIGHT CONDITIONS

Run	Nominal altitude		Nominal Mach number	Nominal $\alpha_t$ , deg	Trim elevator deflection, $\delta_{e,t}$ , deg	Trim bank angle, $\phi_t$ , deg
	m	ft				
1	10 370	34 000	0.71	9.2	-8.13	50
2	10 370	34 000	.66	13.0	-9.74	-60
3 and 4	10 370	34 000	.81	8.2	-7.45	-63
5	10 370	34 000	.78	12.0	-10.3	70

Mass <sup>a</sup>		Moments of inertia <sup>a</sup>							
		$I_X$		$I_Y$		$I_Z$		$I_{XZ}$	
kg	slugs	kg-m <sup>2</sup>	slug-ft <sup>2</sup>	kg-m <sup>2</sup>	slug-ft <sup>2</sup>	kg-m <sup>2</sup>	slug-ft <sup>2</sup>	kg-m <sup>2</sup>	slug-ft <sup>2</sup>
9574.41	656.06	12 500	9200	118 000	86 800	124 000	91 600	4030	2970

<sup>a</sup>Any errors in assuming nominal conditions were no greater than 3 percent of the system uncertainty on the estimated values of the mass and inertias.

TABLE III.- INSTRUMENT RANGES<sup>a</sup>

State	Range
$a_y$	$\pm 0.5g$
V	30.91 to 515.15 m/sec (101.34 to 1689.0 ft/sec)
$\theta$	$\pm 30^\circ$
$\phi$	$\pm 90^\circ$
q	$\pm 20$ deg/sec
r	$\pm 10$ deg/sec
p	$\pm 40$ deg/sec
$\alpha_t$	$-5^\circ$ to $+30^\circ$
$\beta$	$\pm 20^\circ$
h	0 to 21 000 m (0 to 63 000 ft)
$\delta_a$	$-15^\circ$ to $+45^\circ$
$\delta_r$	$\pm 21^\circ$

<sup>a</sup> $v$  was calculated from  $v = V \sin \beta$ . Individual sensors are basically more accurate than 3 percent of full scale; however, because of unknown errors, the effects of incompatibilities between measured states and processing errors, the system accuracy was assumed to be 3 percent of the full-scale range of the instrument for the data used during the extraction procedure.

TABLE IV.- STANDARD DEVIATIONS OF COMPUTED STATES  
FROM MEASURED STATES AT CONVERGENCE.

$\alpha_t$ , deg	M	Standard deviation of -					
		v		p,	r,	$\phi$ ,	$a_y$ ,
		m/sec	ft/sec	rad/sec	rad/sec	rad	g units
9.2	0.71	0.3048	1.0	0.013	0.0041	0.068	0.008
13.0	.66	.4023	1.32	.018	.0031	.031	.013
<sup>a</sup> 8.2	.81	.2804	.92	.016	.0026	.019	.010
<sup>b</sup> 8.2	.81	.3200	1.05	.028	.0022	.036	.019
<sup>a</sup> 12.0	.78	.4755	1.56	.030	.0040	.048	.011

<sup>a</sup>Aileron input.

<sup>b</sup>Rudder input.

TABLE V.- EXTRACTED VALUES OF PARAMETERS AND  
ESTIMATED STANDARD DEVIATIONS

Parameters	Extracted values (and standard deviations) for -			
	$\alpha_t = 9.2^\circ$ M = 0.71	$\alpha_t = 13^\circ$ M = 0.66	$\alpha_t = 8.2^\circ$ M = 0.81 (a)	$\alpha_t = 12^\circ$ M = 0.78 (b)
$C_{Y\beta}$	-0.80 (0.03)	-0.80 (0.04)	-0.80 (0.06)	-0.74 (0.035)
$C_{Yp}$	0.18 (0.09)	0.17 (0.11)	0.37 (0.12)	0.13 (-0.15)
$C_{Yr}$	0.45 (0.45)	0.45 (0.7)	0.45 (0.5)	0.45 (-0.6)
$C_{Y\delta_r}$	0.11 (0.036)	0.06 (0.03)	0.06 (0.026)	0.06 (Fixed)
$C_{l\beta}$	-0.12 (0.003)	-0.112 (0.0013)	-0.14 (0.0015)	-0.11 (0.002)
$C_{lp}$	-0.31 (0.009)	-0.28 (0.0034)	-0.36 (0.004)	-0.22 (0.008)
$C_{lr}$	0.51 (0.027)	0.33 (0.027)	0.37 (0.03)	0.24 (0.02)
$C_{l\delta_r}$	0.032 (0.0024)	0.028 (0.0009)	0.037 (0.00067)	0.03 (Fixed)
$C_{l\delta_a}$	-0.051 (0.0012)	-0.046 (0.0007)	-0.055 (0.00052)	-0.044 (0.0008)
$C_{n\beta}$	0.102 (0.003)	0.103 (0.0018)	0.11 (0.002)	0.095 (0.003)
$C_{np}$	-0.045 (0.001)	-0.036 (0.0045)	-0.06 (0.003)	-0.04 (0.01)
$C_{nr}$	-0.40 (0.026)	-0.62 (0.027)	-0.33 (0.02)	-0.33 (0.02)
$C_{n\delta_r}$	-0.12 (0.0027)	-0.13 (0.0013)	-0.115 (0.0007)	-0.115 (Fixed)
$C_{n\delta_a}$	-0.012 (0.0013)	-0.01 (0.0009)	-0.0084 (0.00079)	-0.0004 (0.001)

<sup>a</sup>The values given were determined by taking a weighted average of the results from runs using aileron inputs only or rudder inputs only.

<sup>b</sup>The vehicle was excited by using an aileron input only.

TABLE VI.- CORRELATION MATRICES FOR EXTRACTED AERODYNAMIC PARAMETERS

(a)  $\alpha_t = 9.2^\circ$  and  $M = 0.71$ , with both aileron and rudder inputs

	$C_{Y_\beta}$	$C_{Y_p}$	$C_{Y_r}$	$C_{Y_{\delta_r}}$	$C_{l_\beta}$	$C_{l_p}$	$C_{l_r}$	$C_{l_{\delta_r}}$	$C_{l_{\delta_a}}$	$C_{n_\beta}$	$C_{n_p}$	$C_{n_r}$	$C_{n_{\delta_r}}$	$C_{n_{\delta_a}}$
$C_{Y_\beta}$	1	0.002	-0.093	-0.2	0.088	-0.035	0.072	0.095	0.064	0.13	0.15	-0.19	-0.19	0.19
$C_{Y_p}$	0.002	1	-0.58	-0.13	-0.034	-0.01	0.015	-0.024	-0.064	-0.060	-0.066	-0.045	0.063	-0.021
$C_{Y_r}$	-0.093	-0.58	1	0.3	0.06	0.03	0.18	0.003	0.046	-0.032	-0.044	-0.16	-0.04	-0.06
$C_{Y_{\delta_r}}$	-0.2	-0.13	0.3	1	0.016	0.006	0.014	0.021	0.025	-0.038	0.009	-0.011	-0.002	-0.02
$C_{l_\beta}$	0.088	-0.034	0.06	0.016	1	0.63	-0.37	-0.46	0.70	0.73	0.51	-0.61	-0.59	0.59
$C_{l_p}$	-0.035	-0.01	0.03	0.006	0.63	1	-0.54	-0.35	0.89	0.40	0.45	-0.36	-0.13	0.44
$C_{l_r}$	0.072	0.015	0.18	0.014	-0.37	-0.54	1	0.50	-0.45	-0.43	-0.35	-0.19	0.13	-0.41
$C_{l_{\delta_r}}$	0.095	-0.024	0.003	0.021	-0.46	-0.35	0.50	1	-0.35	-0.30	0.018	0.025	0.16	-0.10
$C_{l_{\delta_a}}$	0.064	-0.064	0.046	0.025	0.70	0.89	-0.45	-0.35	1	0.46	0.48	-0.44	-0.22	0.40
$C_{n_\beta}$	0.13	-0.060	-0.032	-0.038	0.73	0.40	-0.43	-0.30	0.46	1	0.76	-0.63	-0.76	0.86
$C_{n_p}$	0.15	-0.066	-0.044	0.009	0.51	0.45	-0.35	0.018	0.48	0.76	1	-0.68	-0.53	0.93
$C_{n_r}$	-0.19	-0.045	-0.16	-0.011	-0.61	-0.36	-0.19	0.025	-0.44	-0.63	-0.68	1	0.68	-0.64
$C_{n_{\delta_r}}$	-0.19	0.063	-0.04	-0.002	-0.59	-0.13	0.13	0.16	-0.22	-0.76	-0.53	0.68	1	-0.61
$C_{n_{\delta_a}}$	0.19	-0.021	-0.06	-0.02	0.59	0.44	-0.41	-0.10	0.40	0.86	0.93	-0.64	-0.61	1

TABLE VI.- Continued

(b)  $\alpha_t = 13^\circ$  and  $M = 0.66$ , with both aileron and rudder inputs

	$C_{Y\beta}$	$C_{Yp}$	$C_{Yr}$	$C_{Y\delta_r}$	$C_{l\beta}$	$C_{lp}$	$C_{lr}$	$C_{l\delta_r}$	$C_{l\delta_a}$	$C_{n\beta}$	$C_{np}$	$C_{nr}$	$C_{n\delta_r}$	$C_{n\delta_a}$
$C_{Y\beta}$	1	0.5	0.46	-0.13	0.11	-0.02	0.19	0.13	-0.095	-0.02	0.10	-0.19	-0.10	0.152
$C_{Yp}$	0.5	1	-0.19	-0.33	0.14	0.07	0.16	0.051	-0.065	-0.12	-0.02	-0.16	-0.006	-0.050
$C_{Yr}$	0.46	-0.19	1	0.56	0.07	0.04	0.12	0.08	0.122	-0.03	0.009	-0.11	-0.06	-0.066
$C_{Y\delta_r}$	-0.13	-0.33	0.56	1	0.004	0.014	-0.024	0.024	0.057	-0.027	-0.006	0.04	0.02	-0.018
$C_{l\beta}$	0.11	0.14	0.07	0.004	1	0.6	0.5	-0.09	0.804	0.21	0.18	-0.58	-0.47	0.365
$C_{lp}$	-0.02	0.07	0.04	0.014	0.6	1	0.062	-0.26	0.756	0.10	0.17	-0.33	-0.045	0.085
$C_{lr}$	0.19	0.16	0.12	-0.024	0.5	0.062	1	0.61	-0.176	-0.41	-0.14	-0.63	-0.09	-0.350
$C_{l\delta_r}$	0.13	0.051	0.08	0.024	-0.09	-0.26	0.61	1	-0.378	-0.35	0.10	-0.31	0.06	0.049
$C_{l\delta_a}$	-0.095	-0.065	0.122	0.057	0.804	0.756	-0.176	-0.378	1	0.267	0.150	-0.360	-0.171	-0.045
$C_{n\beta}$	-0.02	-0.12	-0.03	-0.027	0.21	0.10	-0.41	-0.35	0.267	1	0.62	0.11	-0.61	0.897
$C_{np}$	0.10	-0.02	0.009	-0.006	0.18	0.17	-0.14	0.10	0.150	0.62	1	-0.31	-0.44	0.861
$C_{nr}$	-0.19	-0.16	-0.11	0.04	-0.58	-0.33	-0.63	-0.31	-0.360	0.11	-0.31	1	0.5	-0.587
$C_{n\delta_r}$	-0.10	-0.006	-0.06	0.02	-0.47	-0.045	-0.09	0.06	-0.171	-0.61	-0.44	0.5	1	-0.636
$C_{n\delta_a}$	0.152	-0.050	-0.066	-0.018	0.365	0.085	-0.350	0.049	-0.045	0.897	0.861	-0.587	-0.636	1



TABLE VI.- Continued

(c)  $\alpha_t = 8.2^\circ$  and  $M = 0.81$ , with aileron input

	$C_{Y_\beta}$	$C_{Y_p}$	$C_{Y_r}$	$C_{Y_{\delta_r}}$ (a)	$C_{l_\beta}$	$C_{l_p}$	$C_{l_r}$	$C_{l_{\delta_r}}$ (a)	$C_{l_{\delta_a}}$	$C_{n_\beta}$	$C_{n_p}$	$C_{n_r}$	$C_{n_{\delta_r}}$ (a)	$C_{n_{\delta_a}}$
$C_{Y_\beta}$	1	-0.43	-0.003		0.07	-0.09	0.07		-0.07	0.18	0.22	-0.28		0.21
$C_{Y_p}$	-0.43	1	-0.61		0.09	0.05	-0.08		0.03	-0.15	-0.09	0.095		-0.11
$C_{Y_r}$	-0.003	-0.61	1		-0.1	-0.004	0.19		0.03	0.08	-0.09	-0.013		-0.036
$C_{Y_{\delta_r}}$				1										
$C_{l_\beta}$	0.07	0.09	-0.1		1	0.45	-0.39		0.59	0.68	0.49	-0.44		0.65
$C_{l_p}$	-0.09	0.05	-0.004		0.45	1	-0.56		0.86	0.46	0.36	-0.45		0.40
$C_{l_r}$	0.07	-0.08	0.19		-0.39	-0.56	1		-0.42	-0.33	-0.53	0.24		-0.41
$C_{l_{\delta_r}}$								1						
$C_{l_{\delta_a}}$	-0.07	0.03	0.03		0.59	0.86	-0.42		1	0.43	0.32	-0.39		0.33
$C_{n_\beta}$	0.18	-0.15	0.08		0.68	0.46	-0.33		0.43	1	0.74	-0.74		0.94
$C_{n_p}$	0.22	-0.09	-0.09		0.49	0.36	-0.53		0.32	0.74	1	-0.83		0.89
$C_{n_r}$	-0.28	0.095	-0.013		-0.44	-0.45	0.24		-0.39	-0.74	-0.83	1		-0.79
$C_{n_{\delta_r}}$													1	
$C_{n_{\delta_a}}$	0.21	-0.11	-0.036		0.65	0.40	-0.41		0.33	0.94	0.89	-0.79		1

<sup>a</sup> $C_{Y_{\delta_r}}$ ,  $C_{l_{\delta_r}}$ , and  $C_{n_{\delta_r}}$  could not be identified since the control disturbance was from an aileron input.

TABLE VI.- Continued

(d)  $\alpha_t = 8.2^\circ$  and  $M = 0.81$ , with rudder input

	$C_{Y_\beta}$	$C_{Y_p}$	$C_{Y_r}$	$C_{Y_{\delta_r}}$	$C_{l_\beta}$	$C_{l_p}$	$C_{l_r}$	$C_{l_{\delta_r}}$	$C_{l_{\delta_a}}$ (a)	$C_{n_\beta}$	$C_{n_p}$	$C_{n_r}$	$C_{n_{\delta_r}}$	$C_{n_{\delta_a}}$ (a)
$C_{Y_\beta}$	1	0.77	0.85	0.46	0.03	-0.09	0.08	0.12		0.09	0.22	0.02	-0.06	
$C_{Y_p}$	0.77	1	0.5	0.17	0.18	0.07	0.17	0.12		-0.07	0.10	-0.08	-0.07	
$C_{Y_r}$	0.85	0.5	1	0.74	-0.04	-0.09	0.02	0.10		0.15	0.19	0.08	-0.03	
$C_{Y_{\delta_r}}$	0.46	0.17	0.74	1	-0.13	-0.14	-0.12	0.02		0.15	0.15	0.16	0.07	
$C_{l_\beta}$	0.03	0.18	-0.04	-0.13	1	0.80	0.87	0.53		-0.10	-0.24	-0.22	-0.09	
$C_{l_p}$	-0.09	0.07	-0.09	-0.14	0.80	1	0.60	0.29		-0.15	-0.05	-0.38	-0.13	
$C_{l_r}$	0.08	0.17	0.02	-0.12	0.87	0.60	1	0.79		-0.28	-0.36	-0.15	0.11	
$C_{l_{\delta_r}}$	0.12	0.12	0.10	0.02	0.53	0.29	0.79	1		-0.19	-0.10	-0.08	0.11	
$C_{l_{\delta_a}}$									1					
$C_{n_\beta}$	0.09	-0.07	0.15	0.15	-0.10	-0.15	-0.28	-0.19		1	0.60	0.62	0.13	
$C_{n_p}$	0.22	0.10	0.19	0.15	-0.24	-0.05	-0.36	-0.10		0.60	1	0.05	-0.15	
$C_{n_r}$	0.02	-0.08	0.08	0.16	-0.22	-0.38	-0.15	-0.08		0.62	0.05	1	0.75	
$C_{n_{\delta_r}}$	-0.06	-0.07	-0.03	0.07	-0.09	-0.13	0.11	0.11		0.13	-0.15	0.75	1	
$C_{n_{\delta_a}}$														1

<sup>a</sup> $C_{l_{\delta_a}}$  and  $C_{n_{\delta_a}}$  could not be identified since the control disturbance was a rudder input.

TABLE VI.- Concluded

(e)  $\alpha_t = 12^\circ$  and  $M = 0.78$ , with aileron input

	$C_{Y\beta}$	$C_{Yp}$	$C_{Yr}$	$C_{Y\delta_r}$ (a)	$C_{l\beta}$	$C_{lp}$	$C_{lr}$	$C_{l\delta_r}$ (a)	$C_{l\delta_a}$	$C_{n\beta}$	$C_{np}$	$C_{nr}$	$C_{n\delta_r}$ (a)	$C_{n\delta_a}$
$C_{Y\beta}$	1	-0.47	0.1		0.03	-0.10	0.05		-0.13	0.03	0.13	-0.11		0.15
$C_{Yp}$	-0.47	1	-0.74		-0.12	0.25	-0.04		-0.03	-0.05	-0.06	0.06		-0.04
$C_{Yr}$	0.1	-0.74	1		0.03	0.07	0.16		0.11	0.07	-0.05	-0.15		-0.03
$C_{Y\delta_r}$				1										
$C_{l\beta}$	0.03	-0.12	0.03		1	0.47	-0.35		0.78	0.43	0.31	-0.37		0.38
$C_{lp}$	-0.10	0.25	0.07		0.47	1	-0.59		0.76	0.25	-0.12	-0.14		0.07
$C_{lr}$	0.05	-0.04	0.16		-0.35	-0.59	1		-0.40	-0.15	-0.18	-0.2		-0.2
$C_{l\delta_r}$								1						
$C_{l\delta_a}$	-0.13	-0.03	0.11		0.78	0.76	-0.40		1	0.28	0.05	-0.26		0.05
$C_{n\beta}$	0.03	-0.05	0.07		0.43	0.25	-0.15		0.28	1	0.57	-0.61		0.88
$C_{np}$	0.13	-0.06	-0.05		0.31	-0.12	-0.18		0.05	0.57	1	-0.71		0.80
$C_{nr}$	-0.11	0.06	-0.15		-0.37	-0.14	-0.2		-0.26	-0.61	-0.71	1		0.64
$C_{n\delta_r}$													1	
$C_{n\delta_a}$	0.15	-0.04	-0.03		0.38	0.07	-0.2		0.05	0.88	0.80	0.64		1

<sup>a</sup> $C_{Y\delta_r}$ ,  $C_{l\delta_r}$ , and  $C_{n\delta_r}$  could not be identified since the control disturbance was an aileron input.

TABLE VII.- PARAMETER VALUES FROM OTHER SOURCES

Parameters	Values from ref. 6		Values obtained using ref. 9
	Case E	Case F	
	$\alpha_t = 8^\circ$	$\alpha_t = 12^\circ$	$\alpha_t = 8.2^\circ$
	M = 0.8	M = 0.8	M = 0.81
$C_{Y_\beta}$	-1.00	-1.00	-0.82
$C_{Y_p}$	0.175	0.18	
$C_{Y_r}$	0.45	0.45	
$C_{Y_{\delta_r}}$	0.39	0.195	
$C_{l_\beta}$	-0.126	-0.092	-0.115
$C_{l_p}$	-0.240	-0.085	-0.336
$C_{l_r}$	0.057	0.070	
$C_{l_{\delta_r}}$	0.014	0.007	
$C_{l_{\delta_a}}$	-0.060	-0.060	
$C_{n_\beta}$	0.143	0.092	0.17
$C_{n_p}$	-0.032	-0.035	
$C_{n_r}$	-0.30	-0.33	-0.17
$C_{n_{\delta_r}}$	-0.104	-0.052	
$C_{n_{\delta_a}}$	-0.004	-0.0005	

TABLE VIII. - PERIODS AND DAMPING RATIOS FOR THE DUTCH ROLL MODE

Case	M	$\alpha_t$ , deg	$P_f$ , sec	$P_c$ , sec	$\zeta_f$	$\zeta_c$	Conditions
A	0.71	9.2	2.2	2.6	0.2	0.23	Parameters extracted from flight data as given in table V.
B	.66	13	2.2	2.5	.2	.23	Parameters extracted from flight data as given in table V.
C	.81	8.2	2.2	2.3	.2	.20	Parameters extracted from flight data as given in table V.
D	.78	12		2.2		.19	Parameters extracted from flight data as given in table V.
E	<sup>a</sup> .8	8	2.2	2.0	.2	.17	Parameters from reference 6 as given in table VII.
F	<sup>a</sup> .8	12		2.2		.12	Parameters from reference 6 as given in table VII.

<sup>a</sup>From reference 6 model.

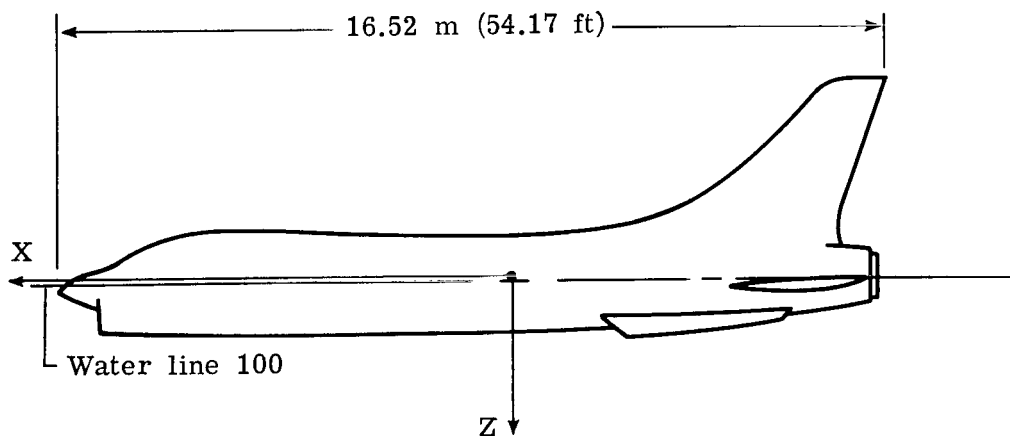
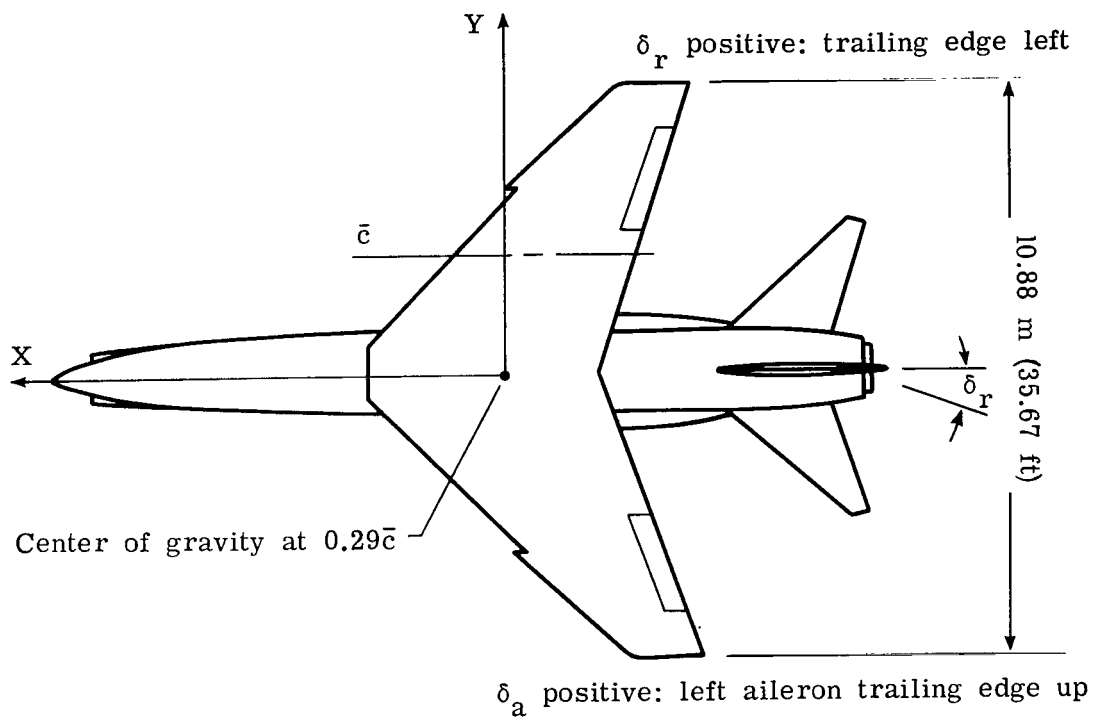


Figure 1.- Sketch of airplane.

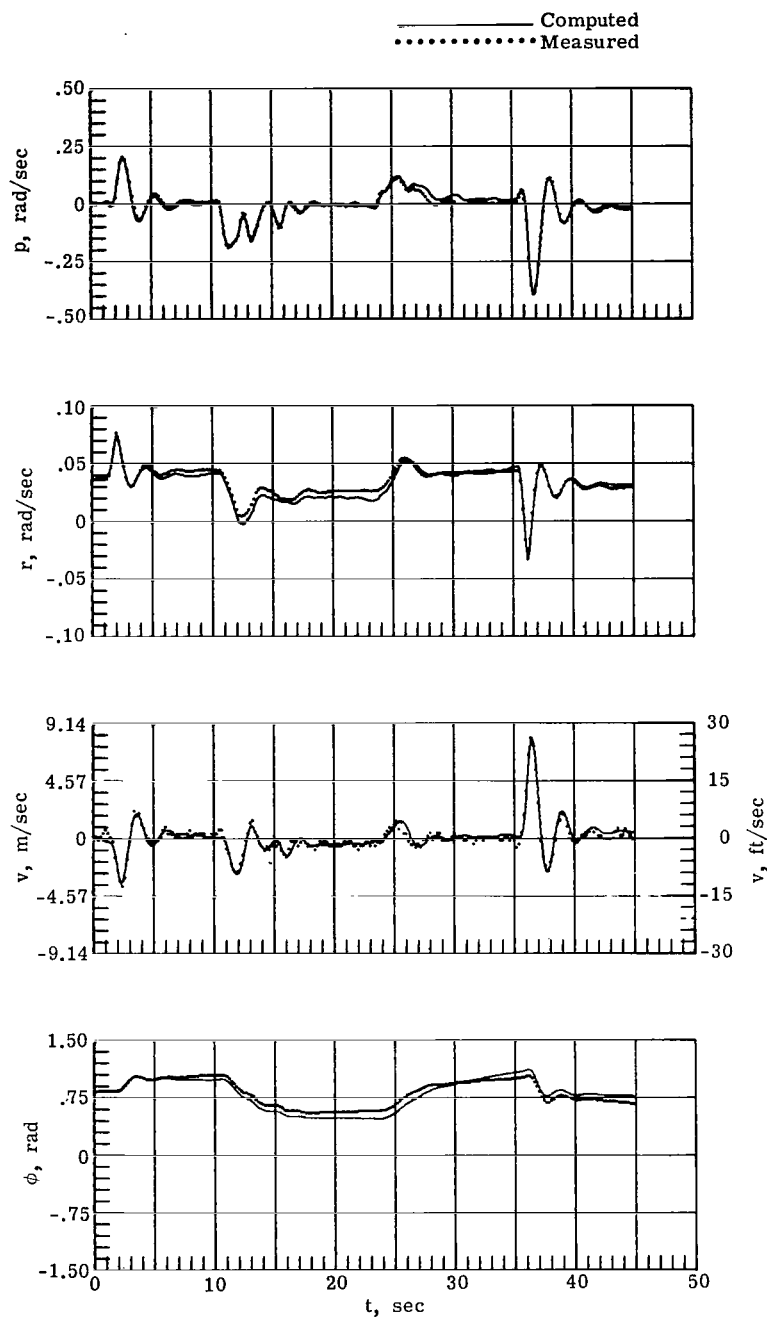


Figure 2.- Comparison of measured data with time histories computed by using parameters given in table V for flight data taken at  $M = 0.71$  and  $\alpha_t = 9.2^\circ$ , with both rudder and aileron inputs.

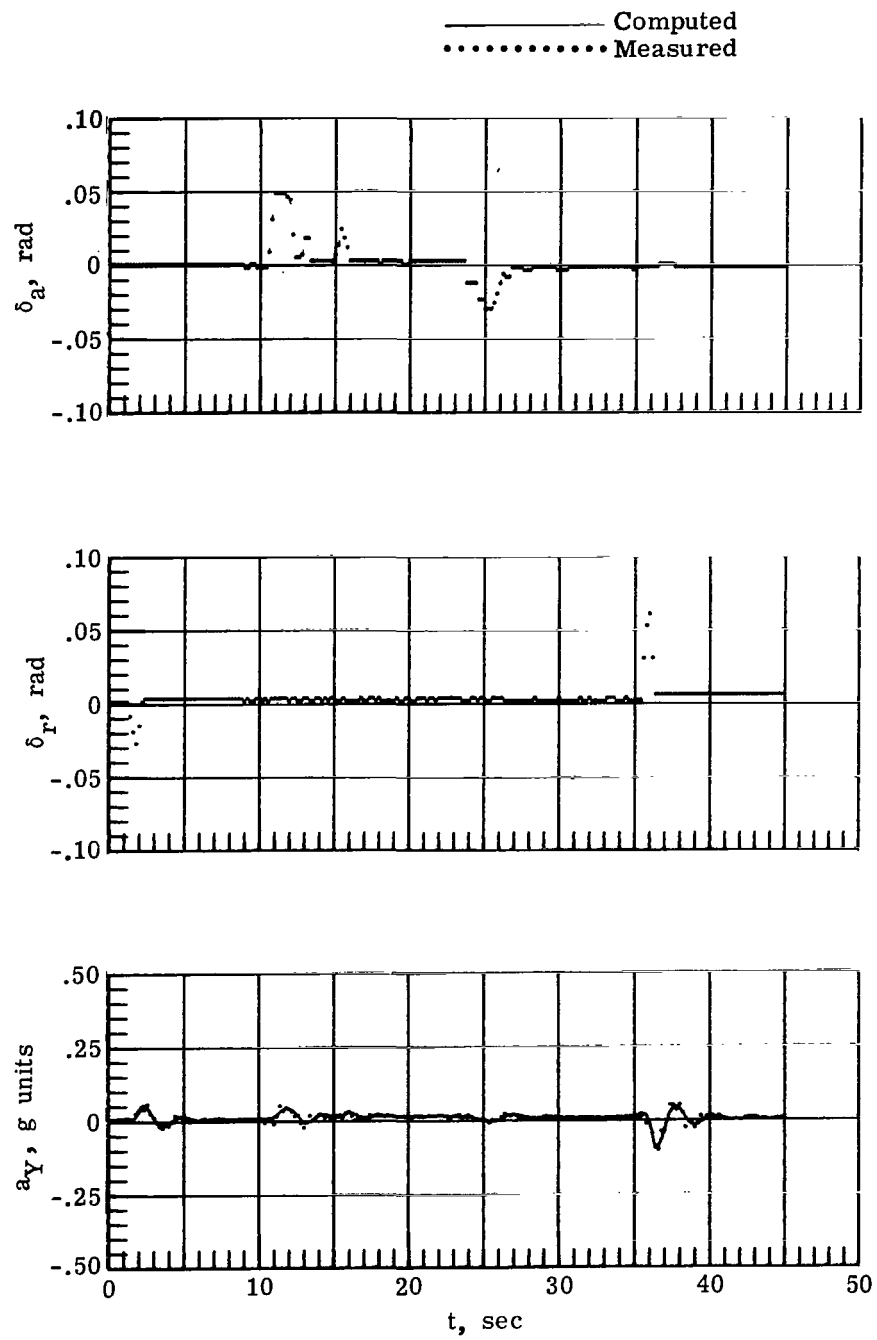


Figure 2.- Concluded.



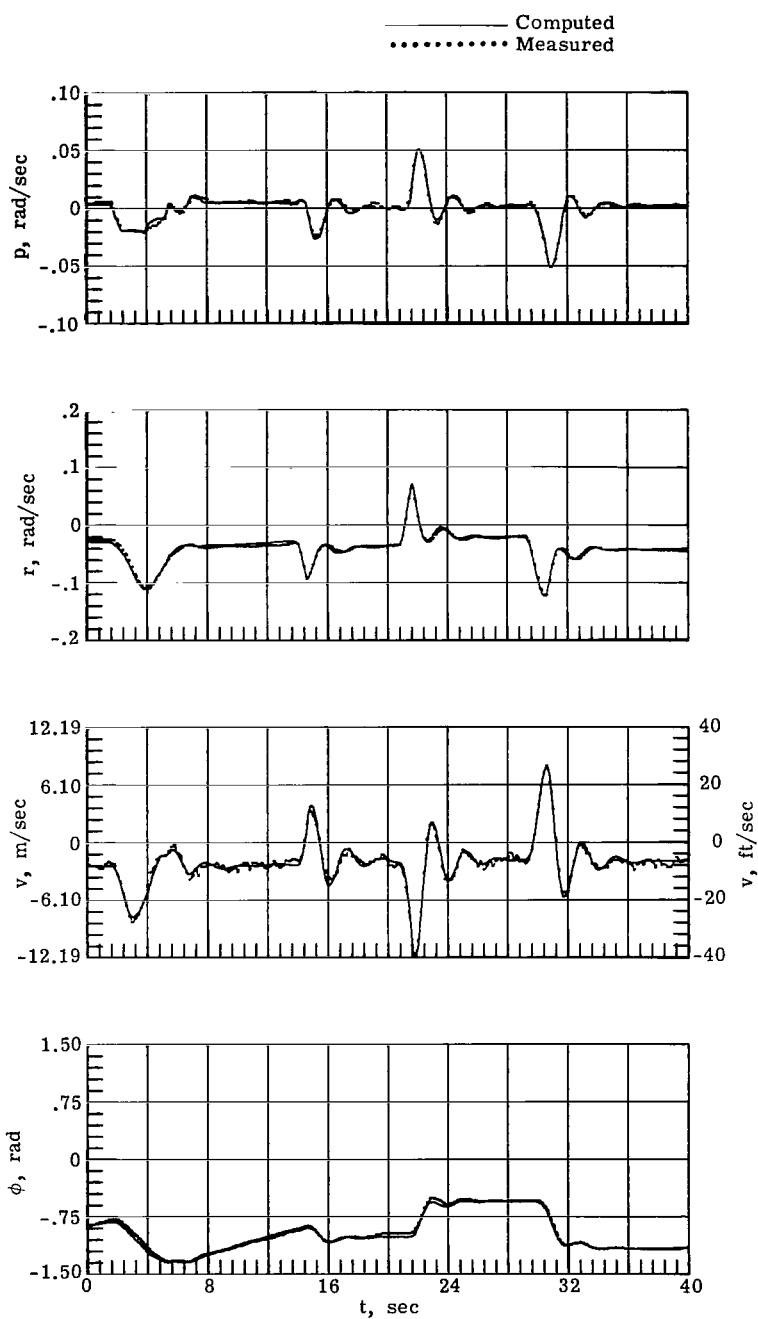


Figure 3.- Comparison of measured data with time histories computed by using parameters given in table V for flight data taken at  $M = 0.66$  and  $\alpha_t = 13^\circ$ , with both aileron and rudder inputs.

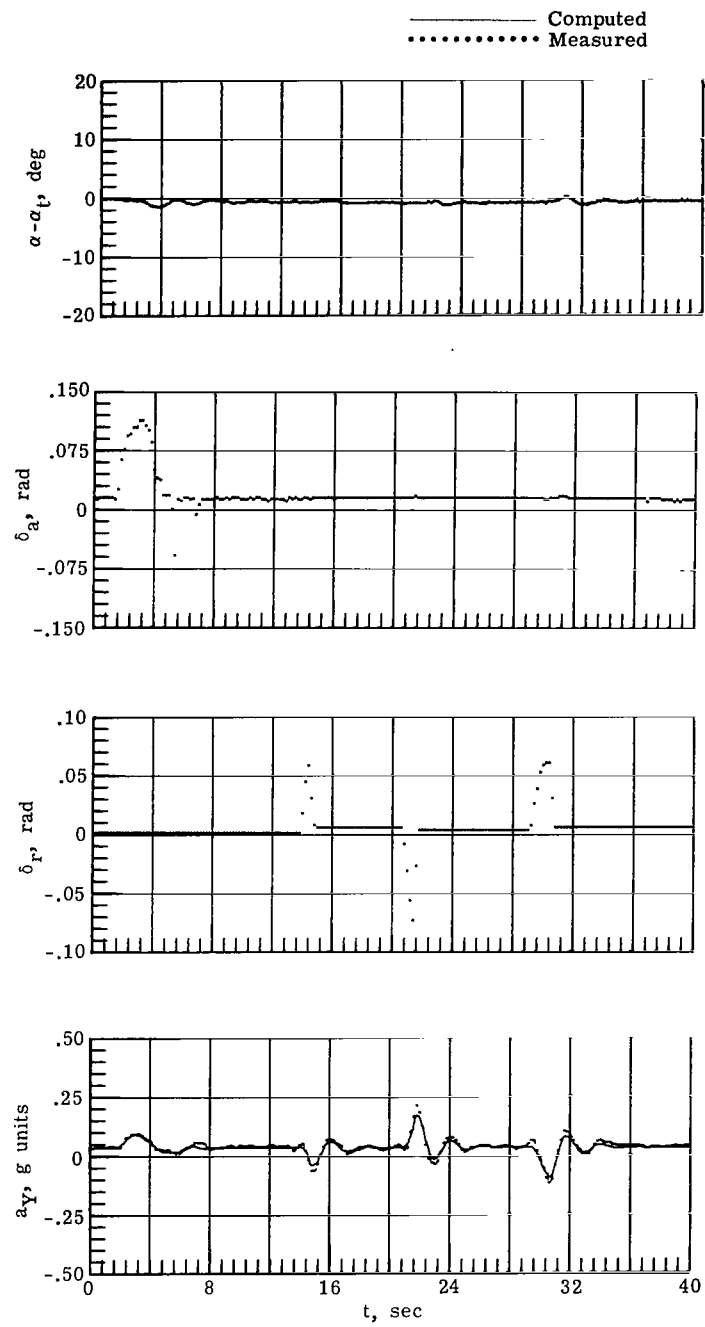


Figure 3.- Concluded.

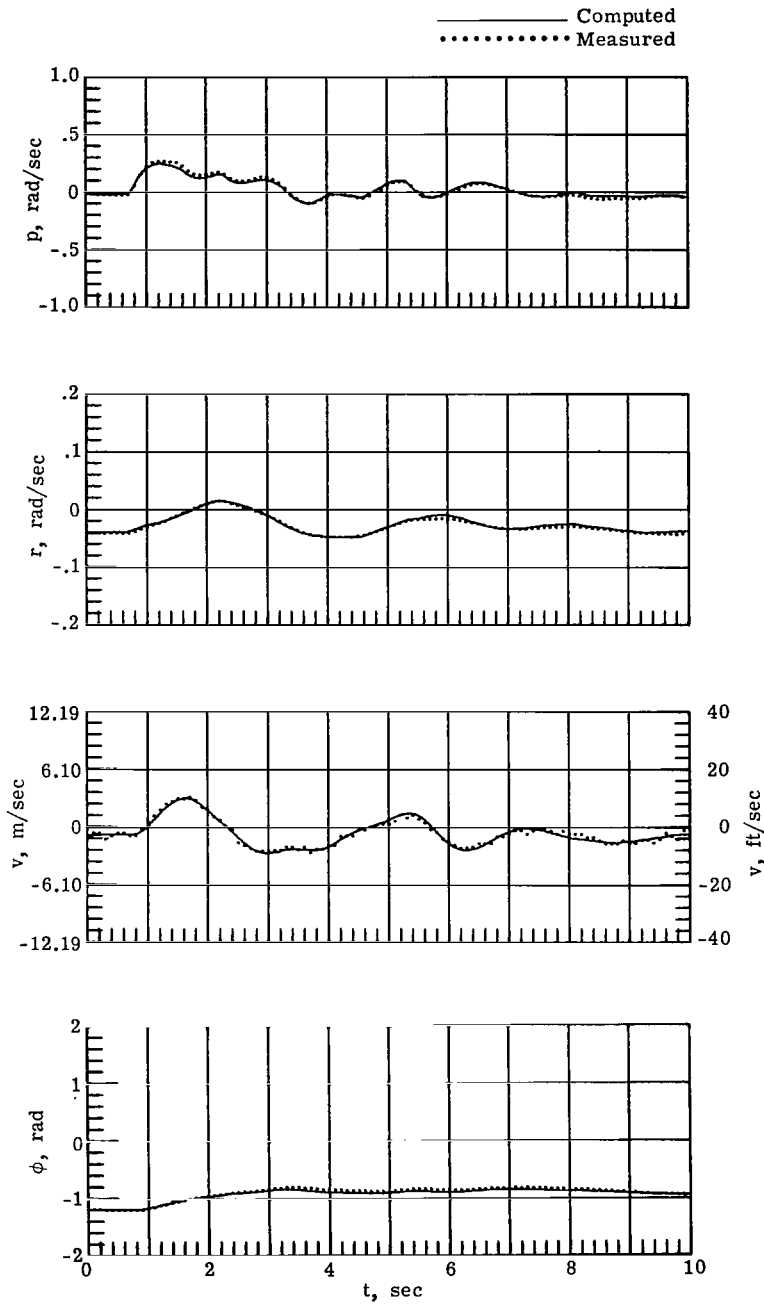


Figure 4.- Comparison of measured data with time histories computed by using parameters given in table V for flight data taken at  $M = 0.81$  and  $\alpha_t = 8.2^\circ$ , with aileron inputs.

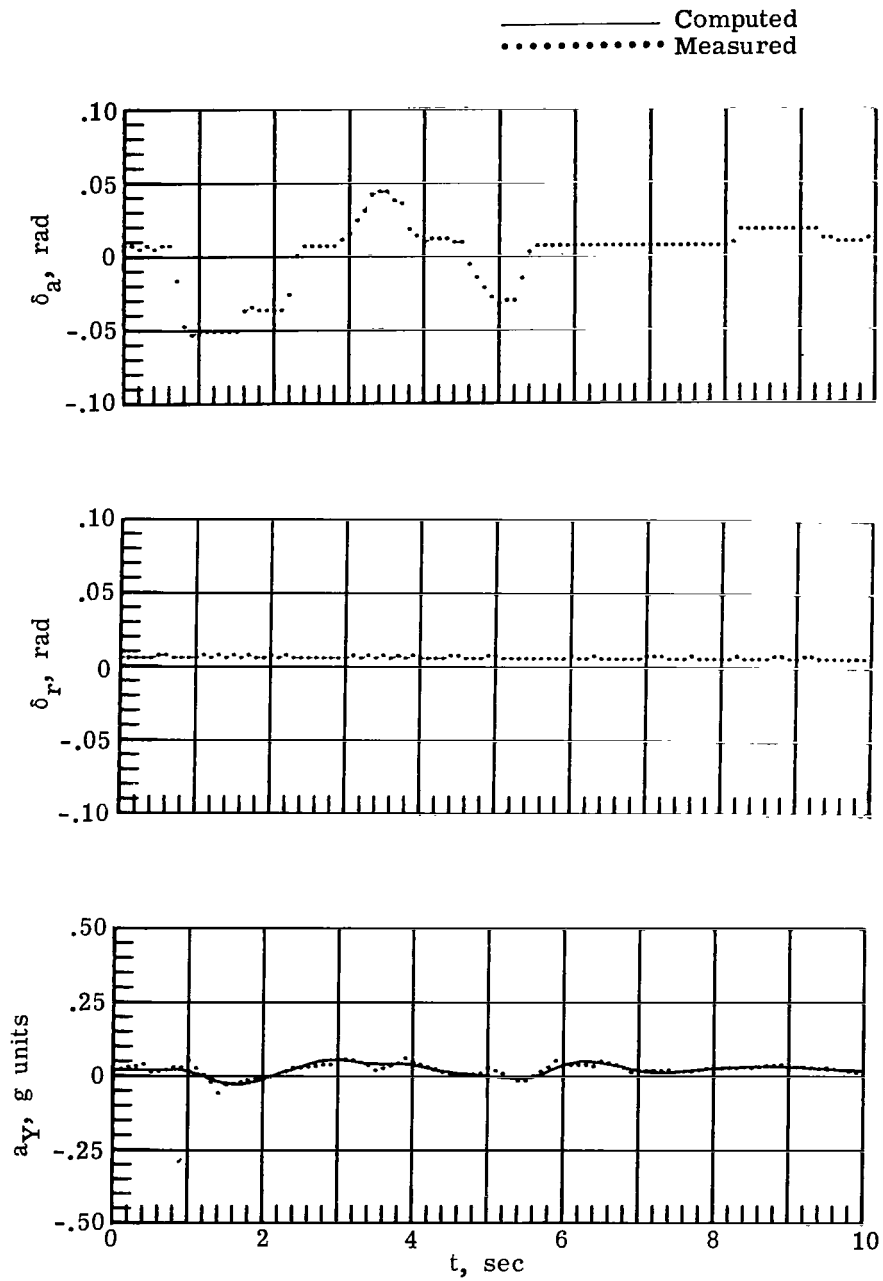


Figure 4.- Concluded.

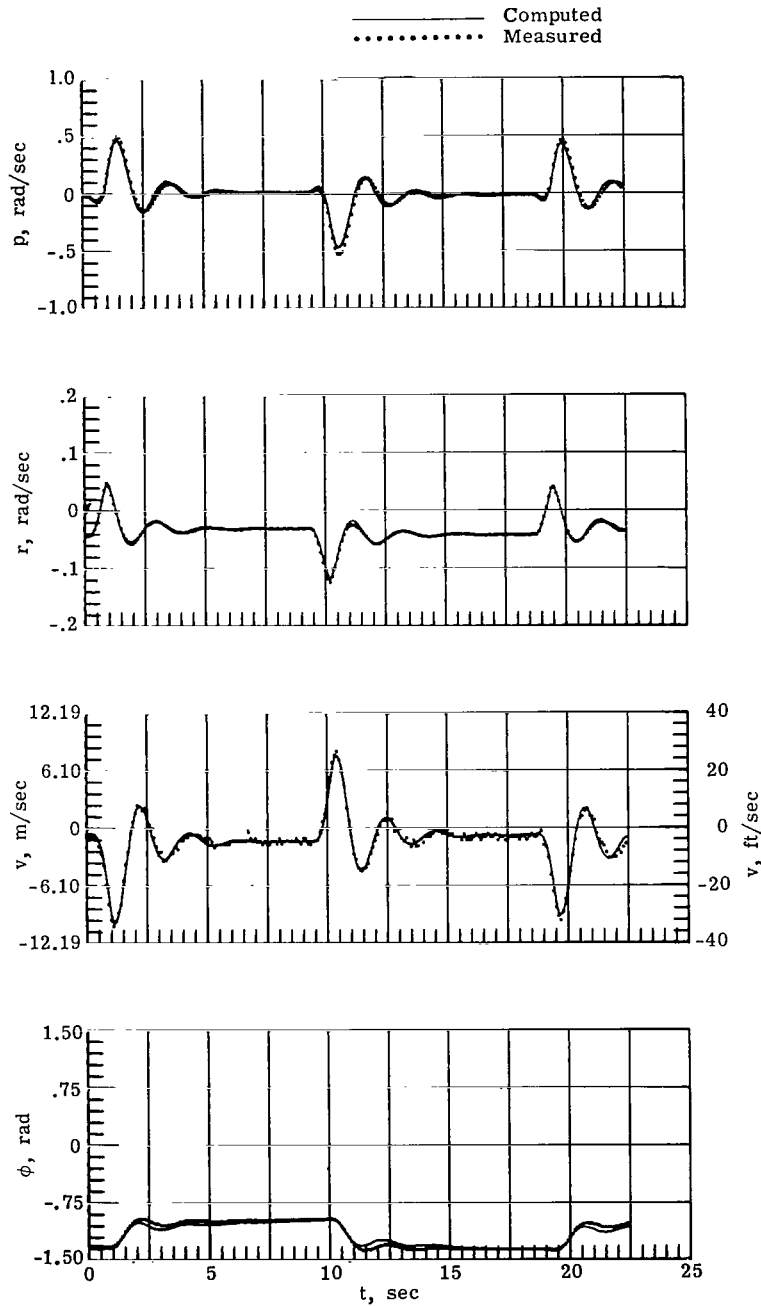


Figure 5.- Comparison of measured data with time histories computed by using parameters given in table V for flight data taken at  $M = 0.81$  and  $\alpha_t = 8.2^\circ$ , with rudder inputs.

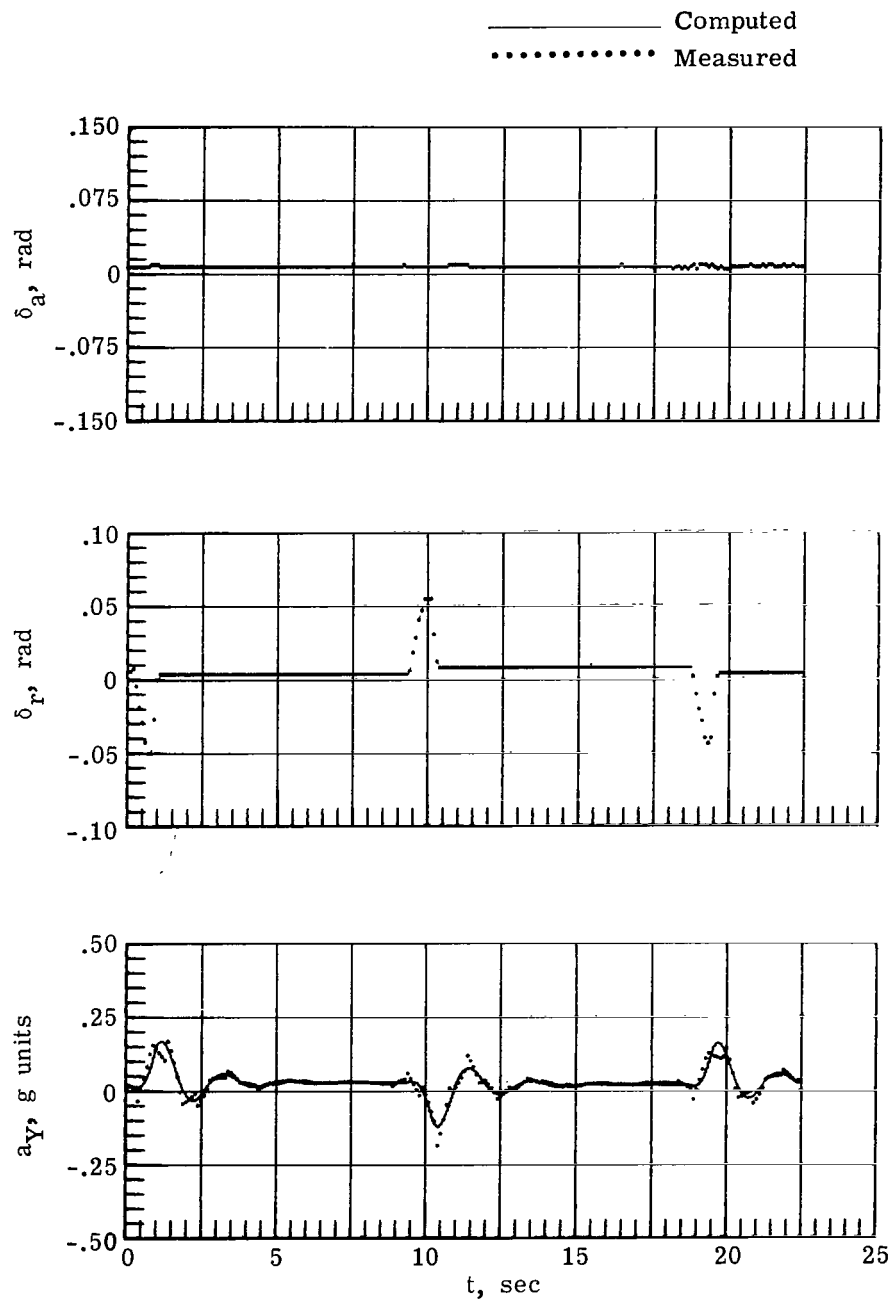


Figure 5.- Concluded.

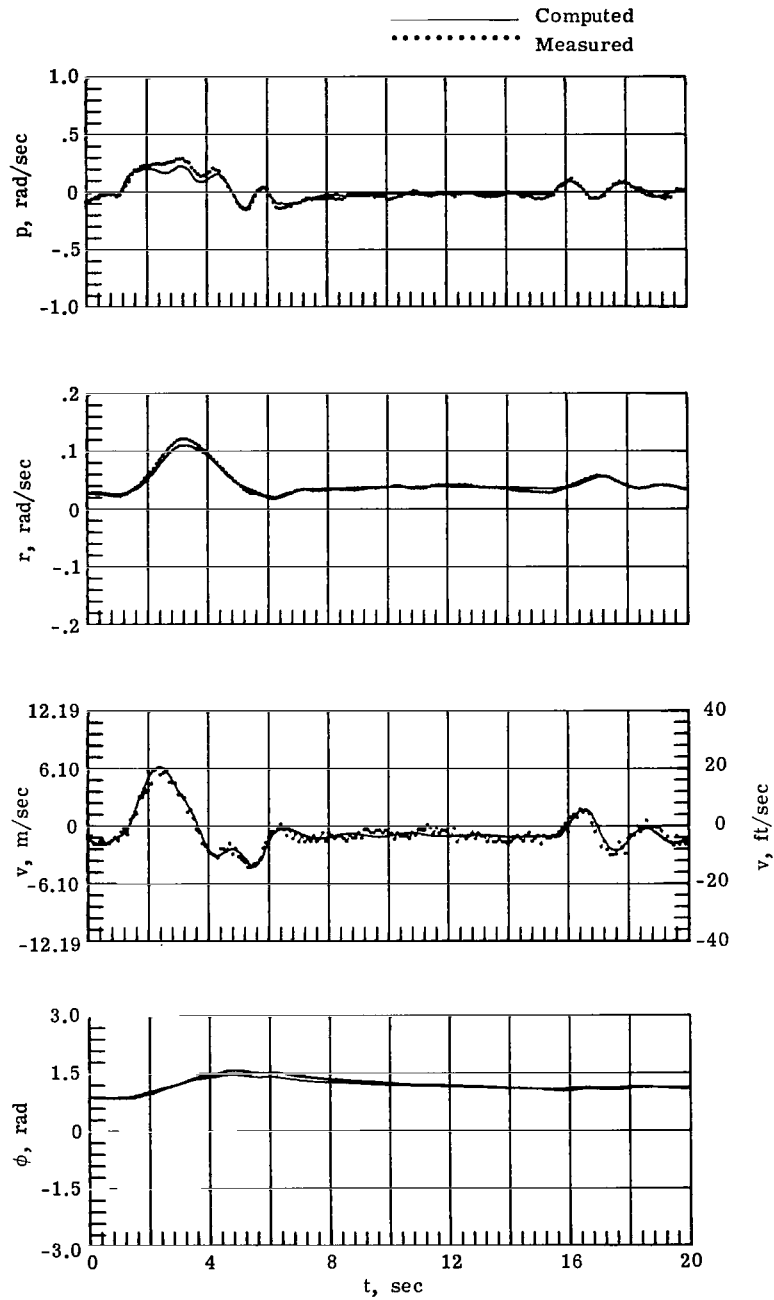


Figure 6.- Comparison of measured data with time histories computed by using parameters given in table V for flight data taken at  $M = 0.78$  and  $\alpha_t = 12^\circ$ , with aileron inputs.

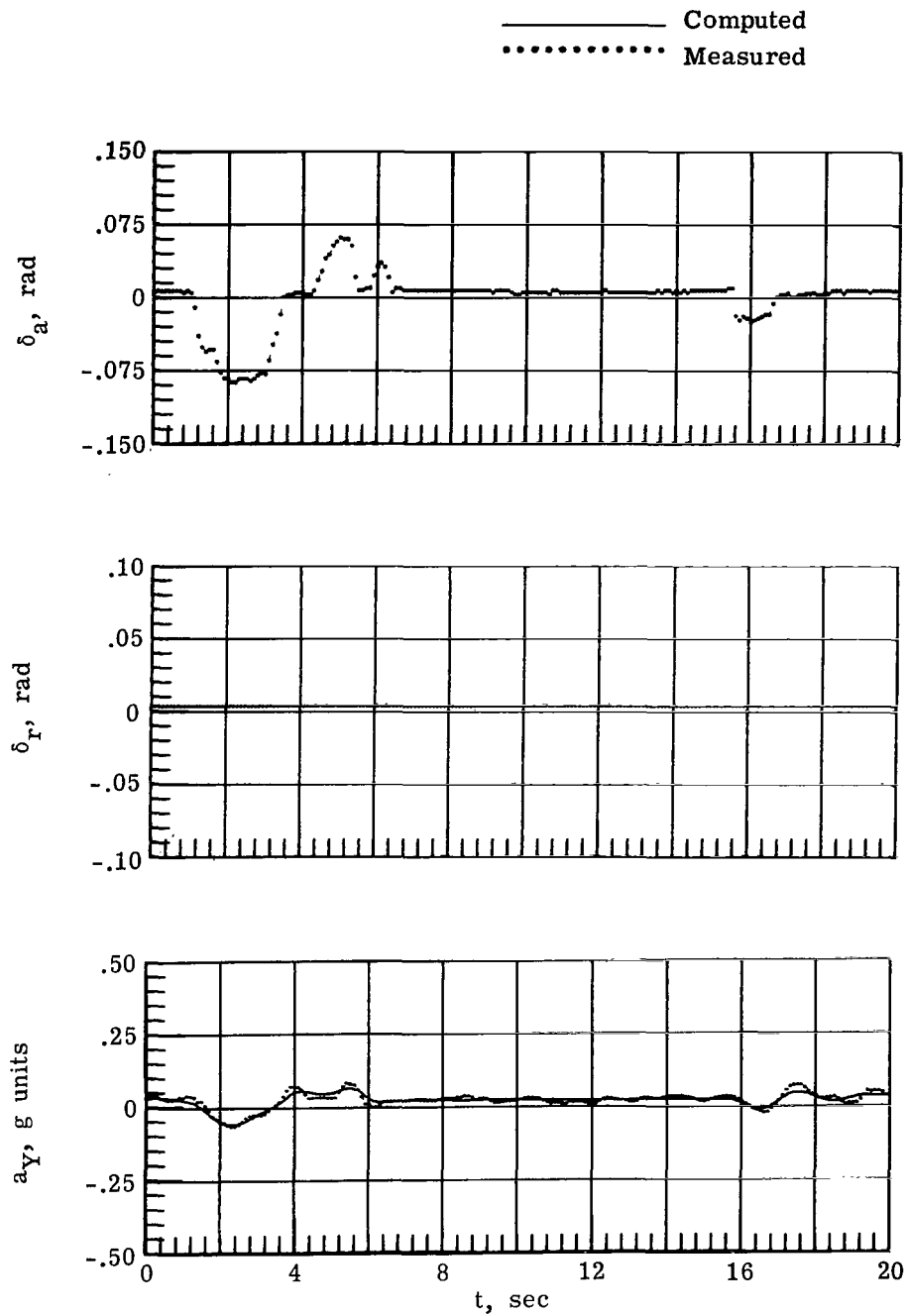


Figure 6.- Concluded.



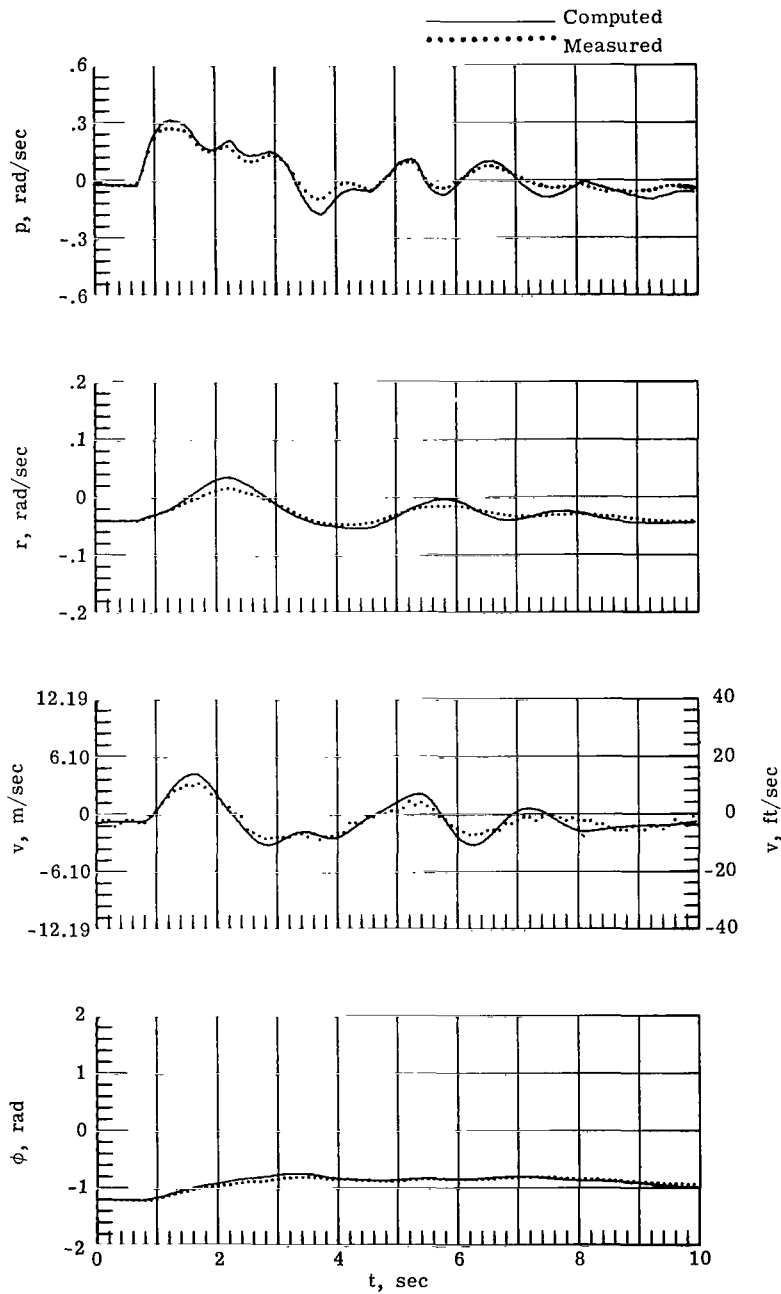


Figure 7.- Comparison of measured data of figure 4 with the time histories computed by using the parameters of reference 6 as given in table VII for Case E.

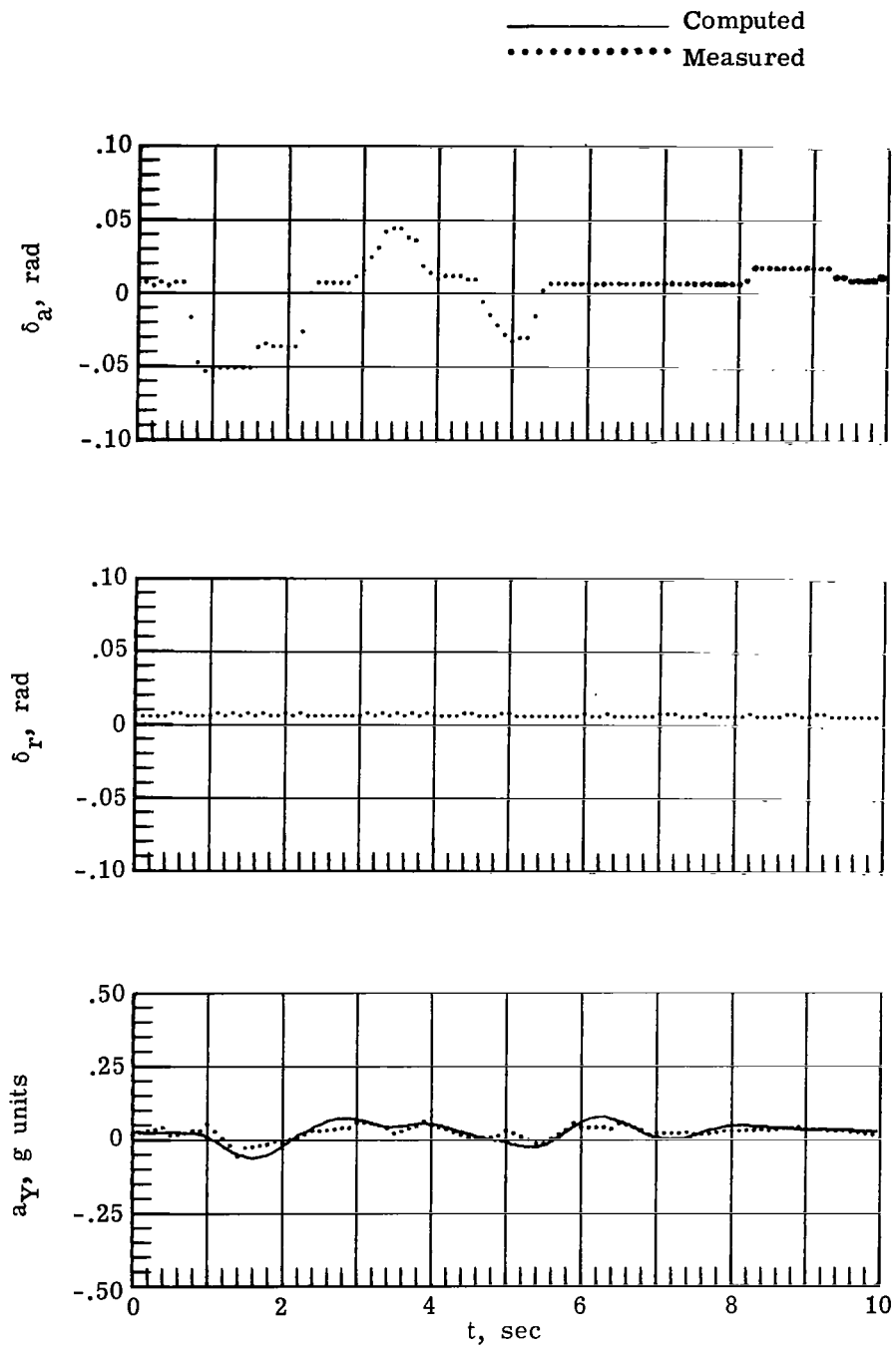


Figure 7.- Concluded.

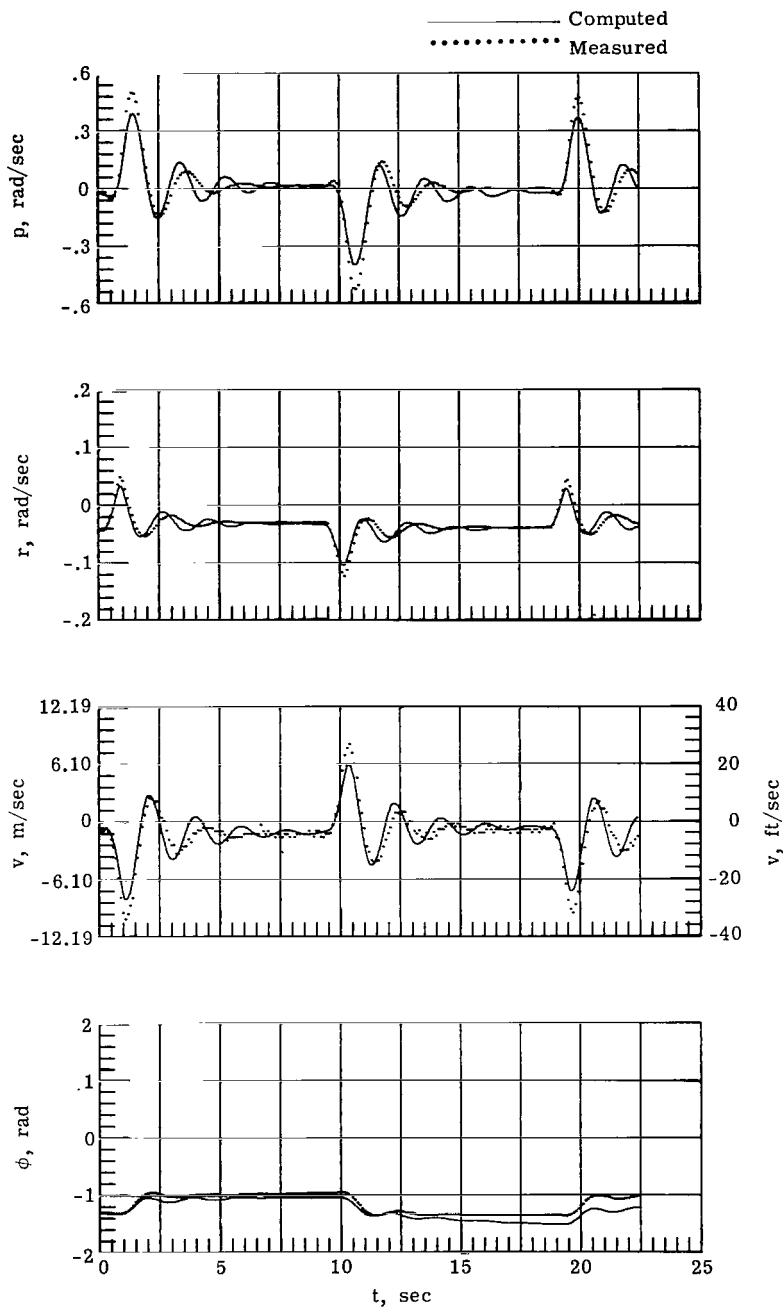


Figure 8.- Comparison of measured data of figure 5 with the time histories computed by using the parameters of reference 6 as given in table VII for Case E.

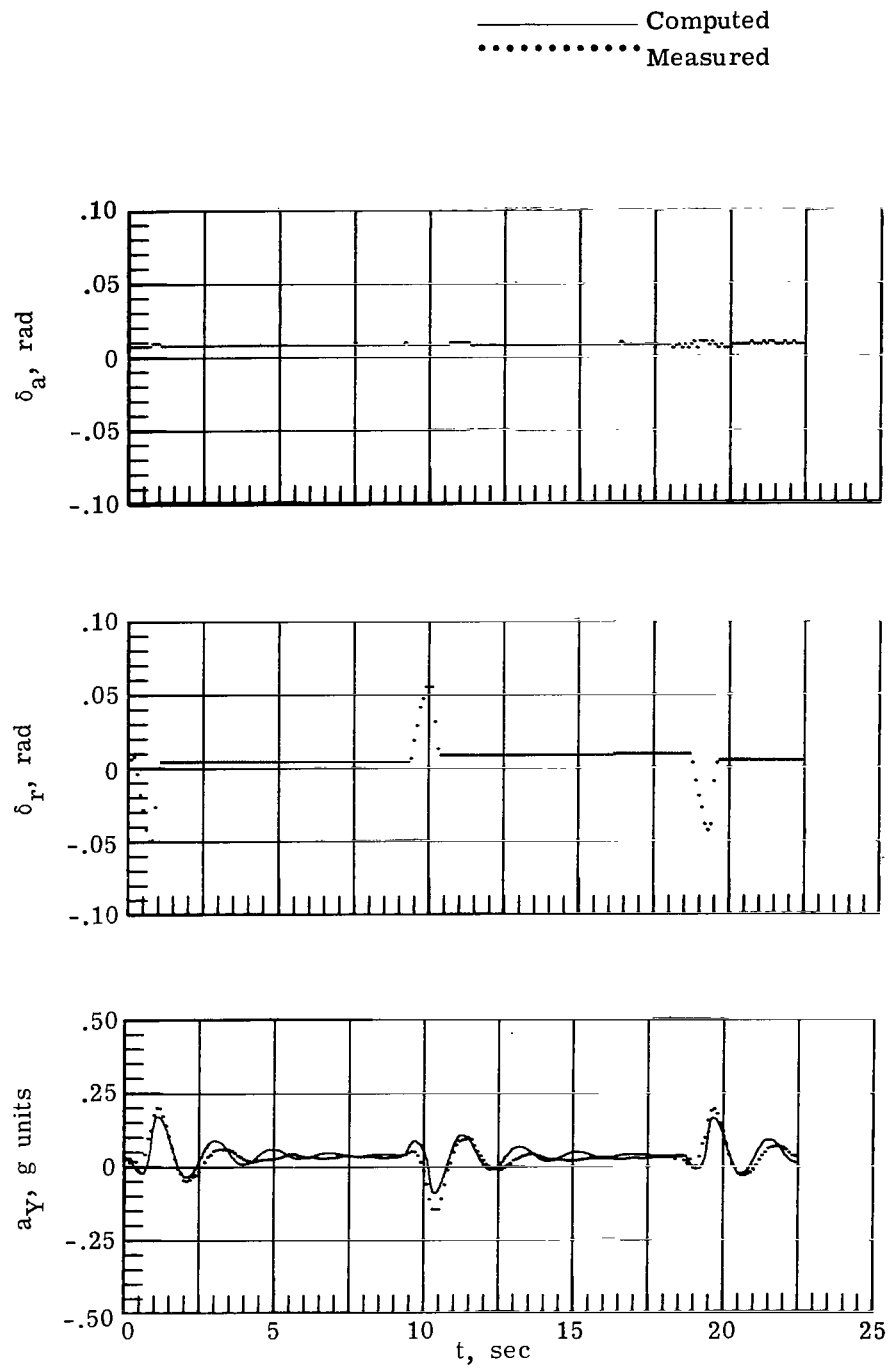


Figure 8.- Concluded.

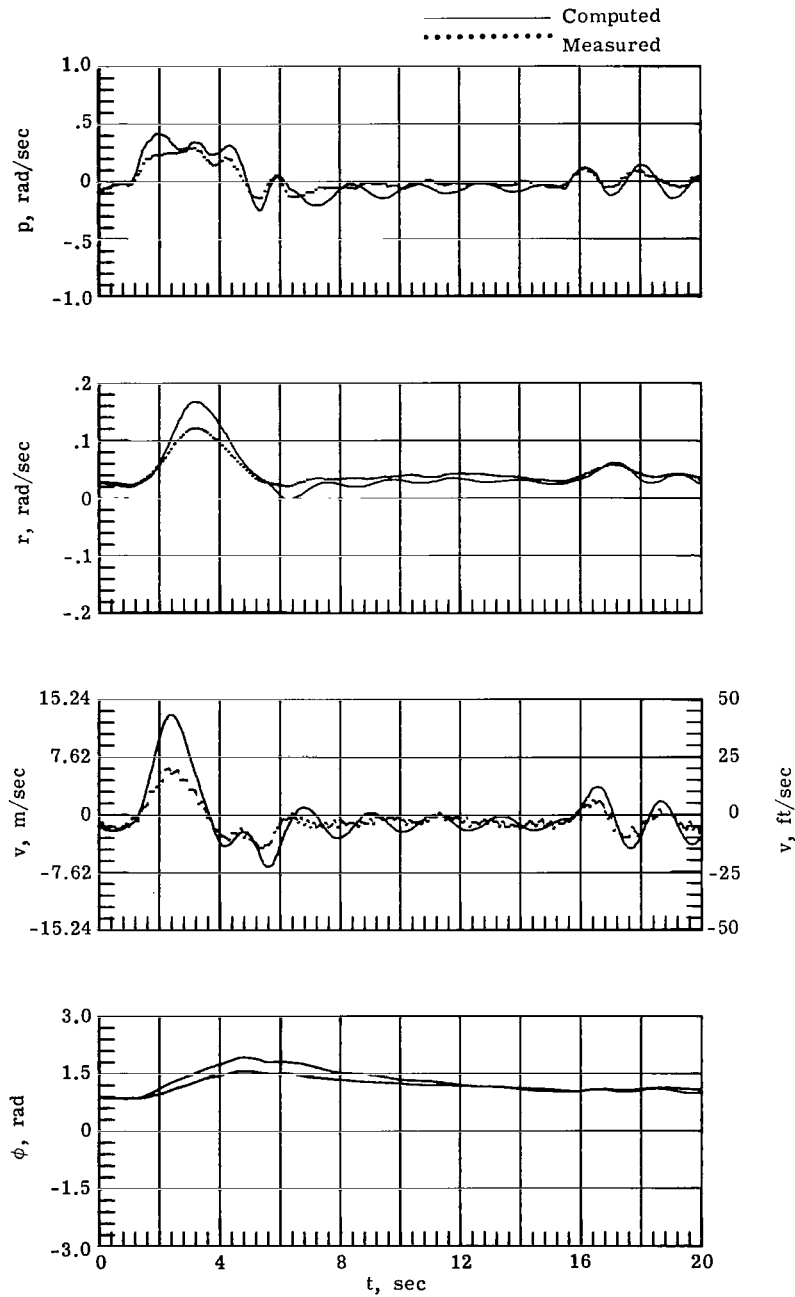


Figure 9.- Comparison of measured data of figure 6 with the time histories computed by using the parameters of reference 6 as given in table VII for Case F.

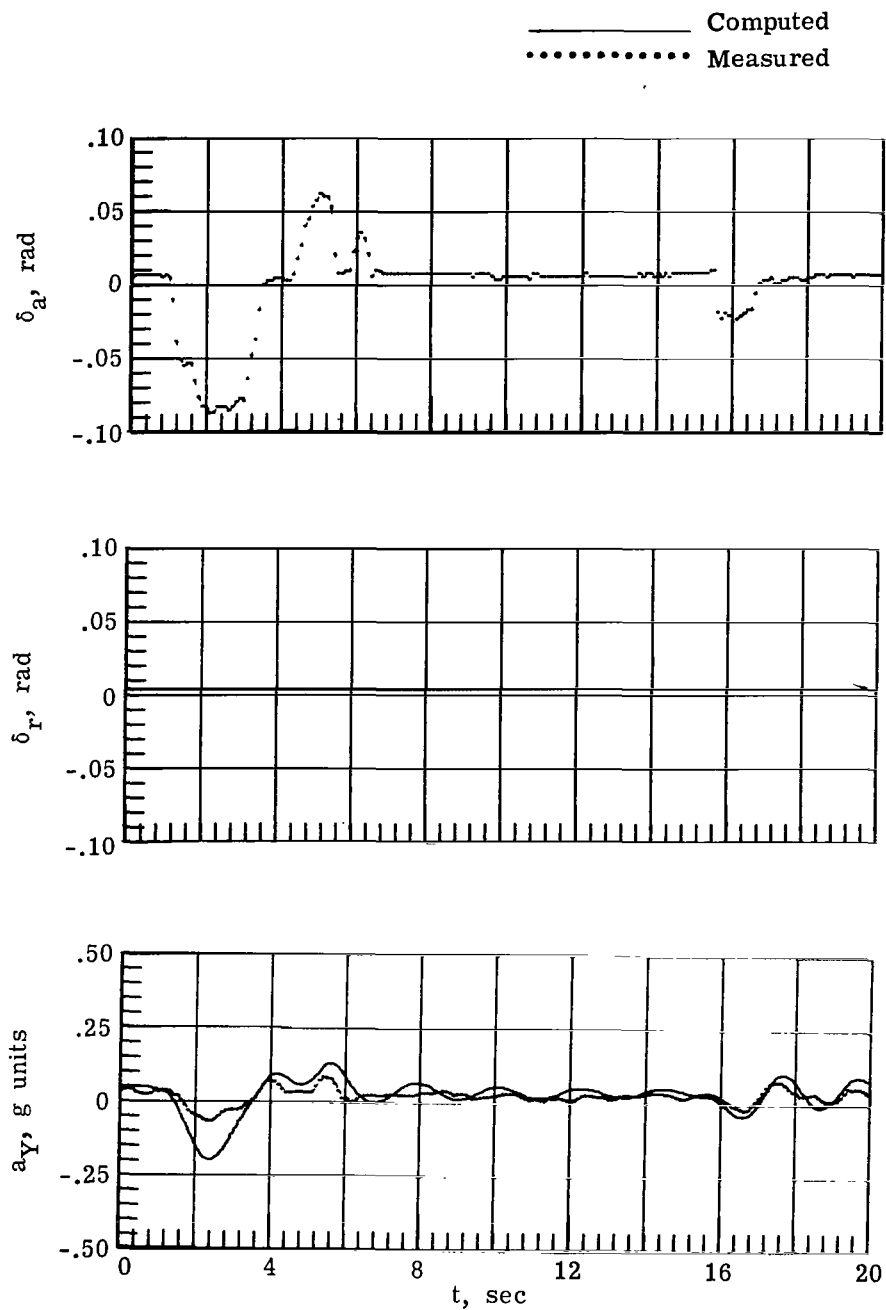


Figure 9.- Concluded.



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